THE ENVIRONMENTAL IMPACT AND SUSTAINABILITY APPLIED GENERAL EQUILIBRIUM (ENVISAGE) MODEL

Version 7.1

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December, 2010

Abstract: This document's main purpose is to provide a full description of the World Bank's global dynamic computable general equilibrium model known as ENVISAGE. ENVISAGE has been developed to assess the interactions between economies and the global environment as affected by human-based emissions of greenhouse gases. At its core, ENVISAGE is a relatively standard recursive dynamic multi-sector multi-region CGE model. It has been complemented by an emissions and climate module that links directly economic activities to changes in global mean temperature. And it incorporates a feedback loop that links changes in temperature to impacts on economic variables such as agricultural yields or damages created by sea level rise. One of the overall objectives of the development of ENVISAGE has been to provide a greater focus on the economics of climate change for a more detailed set of developing countries as well as greater attention to the potential economic damages. The model remains a work in progress as there are several key features of the economics of climate change that are planned to be incorporated in coming months.

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Introduction

The purpose of this document is to provide a complete specification of the equations of the World Bank's ENVIRONMENTAL IMPACT AND SUSTAINABILITY APPLIED GENERAL EQUILIBRIUM (ENVISAGE) MODEL. The ENVISAGE Model is designed to analyze a variety of issues related to the economics of climate change:

- Baseline emissions of CO_2 and other greenhouse gases
- Impacts of climate change on the economy
- Adaptation by economic agents to climate change
- Greenhouse gas mitigation policies—taxes, caps and trade
- The role of land use in future emissions and mitigation
- The distributional consequences of climate change impacts, adaptation and mitigation—at both the national and household level.

ENVISAGE is intended to be flexible in terms of its dimensions. The core database—that includes energy volumes and CO_2 emissions—is the GTAP database, currently version 7.1 with a 2004 base year. The latter divides the world into 112 countries and regions, of which 95 are countries and the other region-based aggregations.[1] The database divides global production into 57 sectors—with extensive details for agriculture and food and energy (coal mining, crude oil production, natural gas production, refined oil, electricity, and distributed natural gas). Annex 8 provides more detail. Due to numerical and algorithmic constraints, a typical model is limited to some 20-30 sectors and 20-30 regions.

This document describes the current version of ENVISAGE, which is still in a developmental stage. This current version includes the following:

- Capital vintage production technology that permits analysis of the flexibility of economies
- A detailed specification of energy demand in each economy, with additions yet to come (see below)
- The ability to introduce future alternative energy (or backstop) technologies
- CO_2 emissions that are fuel and demand specific
- Incorporation of the main Kyoto greenhouse gases (methane, nitrous oxide and the fluoridated gases)
- A flexible system for incorporating any combination of carbon taxes, emission caps and tradable permits
- A simplified climate module that links greenhouse gas emissions to atmospheric concentrations combined with a carbon cycle that leads to radiative forcing and temperature changes.

The future work program includes the following tasks:

1 The countries defined in GTAP cover well over 90 percent of global GDP and population. The country coverage is weakest for Sub-Saharan Africa and the Middle East—though with ongoing work to extend the country coverage.

- Adding a resource depletion module for coal, oil and gas
- Addition of marginal abatement cost curves for the non-CO_2 gases
- Adding a more detailed land-use module
- Adding additional alternative technologies

Model specification

Table 1: Sets used in model definition

Set	***Description***
aa	Armington agents
a	Activities (a subset of *aa*)
i	Produced goods
Manu	Set of manufacturing sectors (subset of *i*, used in definition of numéraire)
fp	Factors of production
l	Labor categories (subset of *fp*)
'*Captl*'	Capital account (subset of *fp*)
'*Landr*'	Land account (subset of *fp*)
'*Natrs*'	Natural resource account (subset of *fp*)
in	Institutions (subset of *a*)
h	Households (subset of *in*)
Gov	Government account (subset of *in*)
Inv	Investment account (subset of *in*)
gy	Government revenue accounts
'*ptax*'	Production tax account (subset of *gy*)
'*dtax*'	Sales tax account on domestically produced goods sold domestically (subset of *gy*)
'*mtax*'	Sales tax account on imported goods (subset of *gy*)
'*ttax*'	Import tariff account (subset of *gy*)
'*etax*'	Export tax account (subset of *gy*)
'*vtax*'	Tax on factors of production account (subset of *gy*)
'*ctax*'	Carbon tax (subset of *gy*)
r	Regions
r'	Alias with *r*
HIC	Set of high-income regions (subset of *r*, used in definition of numéraire)
RSAV	Residual region (subset of *r*, must be of single dimension)

Production block

The ENVISAGE production structure relies on a set of nested constant-elasticity-of-substitution (CES) structures[2]—it has somewhat less flexibility than that developed for the Linkage model, but somewhat more than in the standard GTAP model (see figure 1). Like Linkage, production, in the dynamic version of the model is based on a vintage structure of capital, indexed by *v*. In the standard version, there are two vintages—*Old* and *New*, where *New* is capital equipment that is newly installed at the beginning of the period and *Old* capital is capital greater than a year old. The vintage structure impacts model results through two channels. First, it is typically assumed

[2] Some of the key analytical properties of the CES, and its related constant-elasticity-of-transformation (CET) function, are fully described in Annex 1.

that *Old* capital has lower substitution elasticities than *New* capital. Thus countries with higher savings rates will have a higher share of *New* capital and thus greater overall flexibility. The second channel is through the allocation of capital across sectors. *New* capital is assumed to be perfectly mobile across sectors. *Old* capital is sluggish and released using an upward sloping supply curve. In sectors where demand is declining, the return to capital will be less than the economy-wide average. This is explained in greater detail in the market equilibrium section.

Most of the equations in the production structure are indexed by *v*, i.e. the capital vintage. The exceptions are those where it is assumed that the further decomposition of a bundle are no longer vintage specific—such as the demand for non-energy intermediate inputs. Each production activity is indexed by *a*, and is different from the index of produced commodities, *i* (allowing for the combination of outputs from different activities into a single produced good, for example electricity).

(P-1) $$VA_{r,a,v} = \alpha^{va}_{r,a,v}\left(\delta^{cd}_{r,a}\lambda^{v}_{r,a,v}\right)^{\sigma^p_{r,a,v}-1}\left(\frac{PXv_{r,a,v}}{PVA_{r,a,v}}\right)^{\sigma^p_{r,a,v}} XPv_{r,a,v}$$

(P-2) $$ND_{r,a} = \sum_v \alpha^{nd}_{r,a,v}\left(\delta^{cd}_{r,a}\lambda^{n}_{r,a,v}\right)^{\sigma^p_{r,a,v}-1}\left(\frac{PXv_{r,a,v}}{PND_{r,a}}\right)^{\sigma^p_{r,a,v}} XPv_{r,a,v}$$

(P-3) $$PXv_{r,a,v} = \frac{1}{\delta^{cd}_{r,a}}\left[\alpha^{va}_{r,a,v}\left(\frac{PVA_{r,a,v}}{\lambda^{v}_{r,a,v}}\right)^{1-\sigma^p_{r,a,v}} + \alpha^{nd}_{r,a,v}\left(\frac{PND_{r,a}}{\lambda^{n}_{r,a,v}}\right)^{1-\sigma^p_{r,j}}\right]^{1/(1-\sigma^p_{r,a,v})}$$

(P-4) $$PX_{r,a} = (1+\pi_{r,a})\frac{\sum_v PXv_{r,a,v}XPv_{r,a,v}}{XP_{r,a}}$$

(P-5) $$\Pi_{r,a} = \pi_{r,a}\sum_v PXv_{r,a,v}XPv_{r,a,v}$$

(P-6) $$PP_{r,a} = (1+\tau^p_{r,a})PX_{r,a} + \tau^x_{r,a}$$

Equations (P-1) and (P-2) are derived demands for two bundles, one designated as aggregate value added, *VA*, though it also includes energy demand that is linked to capital, and aggregate intermediate demand, *ND*, a bundle that excludes energy. Both are shares of output by vintage, *XPv*, with the shares being price sensitive with respect to the ratio of the vintage-specific unit cost, *PXv*, and the component prices, respectively *PVA* and *PND*. The equations allow for technological change embodied in the λ parameters that are allowed to be node-specific. For uniform technological change, the two parameters can be subject to the same percentage change. Both productivity factors are impacted by the same damage adjustment, δ^{cd}, which is region and sector specific and depends on climate change.[3] Equation (P-3) defines the vintage-specific unit

[3] Discussed further below.

cost, *PXv*. Almost all CES price equations are based on the dual cost function instead of the aggregate cost or revenue formulation. The unit cost function includes the effects of productivity improvement and damages. To the extent climate leads to damages, δ^{cd} drops below its initial level of 1, raising unit cost, all else equal. Equation (P-4) determines the aggregate unit cost, *PX*, the weighted average of the vintage-specific unit costs with the weights given by the vintage-specific output levels. The model allows for a markup, π, to unit cost that is normally exogenous and initialized at 0. The revenue generated by the markup, Π, is defined in equation (P-5). Equation (P-6) determines the final market price for output, *PP*, that is equal to the unit cost augmented by the output tax (or subsidy), τ^p. The equivalence of the tax-adjusted unit cost to the output price is an implication of assuming constant-returns-to-scale technology and perfect competition (and/or the presence of a fixed markup). The production price can also be adjusted by a volume only tax (or an excise tax), represented by τ^x.

The subsequent production nest decomposes the *VA* bundle (value added and energy) into non-capital factors of production on the one hand, *XF*, indexed by *fpx*[4], and the capital/energy bundle on the other hand, *KE*. The key substitution elasticity is given by σ^v. An elasticity of 1 implies a Cobb-Douglas technology.[5] Equation (P-7) determines the demand for non-capital factors (unskilled and skilled labor, land, and a sector-specific factor if it exists).[6] Factor productivity is given by the λ factor. The productivity factor is 'climate sensitive', in other words, it is a combination of a baseline growth assumption that is also sensitive to changes in global climate as measured by the change in global mean temperature. This is explained further below. Equation (P-8) determines demand for the capital/energy bundle, *KE*. The final equation in this nest (P-9) defines the unit price of the value added cum energy bundle, *PVA*.

(P-7) $$XF^d_{r,fpx,a} = \sum_v \alpha^f_{r,fpx,a,v}\left(\lambda^{gf}_{r,fpx,a}\right)^{\sigma^v_{r,a,v}-1}\left(\frac{PVA_{r,a,v}}{PF_{r,fpx,a}}\right)^{\sigma^v_{r,a,v}} VA_{r,a,v}$$

(P-8) $$KE_{r,a,v} = \alpha^{ke}_{r,a,v}\left(\frac{PVA_{r,a,v}}{PKE_{r,a,v}}\right)^{\sigma^v_{r,a,v}} VA_{r,a,v}$$

(P-9) $$PVA_{r,a,v} = \left[\sum_{fpx}\alpha^f_{r,fpx,a,v}\left(\frac{PF_{r,fpx,a}}{\lambda^{gf}_{r,fpz,a}}\right)^{1-\sigma^v_{r,a,v}} + \alpha^{ke}_{r,a,v} PKE^{1-\sigma^v_{r,a,v}}_{r,a,v}\right]^{1/(1-\sigma^v_{r,a,v})}$$

The next nest is a decomposition of the capital/energy bundle, *KE*, into demand for capital (by vintage) and an energy bundle. Equation (P-10) defines the demand for capital by vintage, *KV*. The substitution elasticity is given by σ^{ke}. Equation (P-11) determines the demand for the energy

4 The set *fp* indexes all factors of production, the subset *fpx* excludes capital.

5 In the GAMS implementation of the model, a Cobb-Douglas technology is approximated by an elasticity of 1.01.

6 The model implementation allows for a scale factor (`phiw`) that is used to scale factor prices. This can help with numerical problems.

bundle, *XNRG*. The latter is indexed by *eb*, a special set that indexes all energy bundles. There is a set mapping that has a one-to-one correspondence between the given activity *a* and a specific item in *eb*. The reason for this is to simplify the code for disaggregating the energy bundles across agents in the economy and is described further below. Equation (P-12) defines the price of the *KE* bundle, *PKE*.

(P-10) $$KV_{r,a,v} = \alpha^{f}_{r,Captl,a,v}\left(\lambda^{gf}_{r,Captl,a,v}\right)^{\sigma^{ke}_{r,a,v}-1}\left(\frac{PKE_{r,a,v}}{PKV_{r,a,v}}\right)^{\sigma^{ke}_{r,a,v}} KE_{r,a,v}$$

(P-11) $$XNRG_{r,eb,v} = \alpha^{ep}_{r,a,v}\left(\frac{PKE_{r,a,v}}{PNRG_{r,eb,v}}\right)^{\sigma^{ke}_{r,a,v}} KE_{r,a,v}$$

(P-12) $$PKE_{r,a,v} = \left[\alpha^{f}_{r,Captl,a,v}\left(\frac{PKV_{r,a,v}}{\lambda^{gf}_{r,Captl,a}}\right)^{1-\sigma^{ke}_{r,a,v}} + \alpha^{ep}_{r,a,v} PNRG^{1-\sigma^{ke}_{r,a,v}}_{r,eb,v}\right]^{1/(1-\sigma^{ke}_{r,a,v})}$$

The final node in the production nest is the decomposition of aggregate demand for non-fuel intermediate goods, *ND*. For the moment, we are assuming a standard Leontief technology (although allowing for the possibility of substitution across inputs.[7] Equation (P-13) determines the demand for the (Armington) intermediate demand for non-fuel inputs, XA_n, with the substitution elasticity given by σ^n. The relevant price is the agent (or activity) specific Armington price, PA^a. The latter will be a composite price of domestic and imported goods, augmented by domestic taxes and, in mitigation scenarios, with a tax linked to emissions (described below). The model allows for input-specific efficiency improvements as encapsulated by the λ^{nd} parameter. Equation (P-14) provides the price of the aggregate ND bundle.

(P-13) $$XA_{r,n,a} = io_{r,n,a,Old}\left(\lambda^{nd}_{r,n,a}\right)^{\sigma^{n}_{r,a}-1}\left(\frac{PND_{r,a}}{PA^{a}_{r,n,a}}\right)^{\sigma^{n}_{r,a}} ND_{r,a}$$

(P-14) $$PND_{r,a} = \left[\sum_{n\in nnrg} io_{r,n,a,Old}\left(\frac{PA^{a}_{r,n,a}}{\lambda^{nd}_{r,n,a}}\right)^{1-\sigma^{n}_{r,a}}\right]^{1/(1-\sigma^{n}_{r,a})}$$

This ends the description of the production structure, though there is a further decomposition of XA_n, i.e. the non-fuels intermediate Armington demand, and the energy bundle. The latter is decomposed across fuels, and then finally decomposed as an Armington good.

[7] The Linkage model has a different production structure for crops, livestock and all other goods allowing for more complex interactions between agricultural inputs, for example fertilizers and feed, and the factors of production.

Income block

The model has six indirect tax streams and one direct-tax stream:

1. The output tax, τ^p imposed on the aggregate price of output, *PX*, with an additional excise tax, τ^x, in some circumstances.
2. A sales tax on sales of domestic Armington goods, τ^{Ap}, which is agent specific and imposed on the economy-wide price of domestic goods, *PA*.[8]
3. Bilateral import tariff, τ^m, imposed on the landed (or CIF) price of imports, *WPM*. The model also allows for homogeneous goods, in which case the tariff represents a wedge between the world price and the domestic price.
4. Bilateral export tax (or subsidy), τ^e, imposed on the producer price of exports, *PE*. In the case of a homogeneous commodity, the export tax represents the wedge between world prices and domestic prices.
5. Taxes on the factors of production, τ^v, imposed on the market-clearing price of factors, *NPF*.
6. Taxes on emissions, τ^e, imposed on the Armington consumption of goods.

Equations (Y-1) through (Y-6) correspond to the aggregate revenues generated by each of the six indirect taxes. The notation for the variables not already described will be given below. One important observation concerns the bilateral trade variables. These are always indexed as (*r*, *r*', *i*) where *r* is the country of origin (the exporter), *r*' the country of destination (the importer) and *i* is the sector index. This explains the switch in the indices equations (Y-3) and (Y-4) where *WTF* corresponds to the bilateral trade flow from region *r* to region *r*'. Carbon tax revenues will be a function of the parameter ϕ that allows for full or partial participation—including full exclusion—by agent. Equation (Y-7) defines the aggregate revenue from taxing household income. Fiscal closure will be discussed below.

8 The model allows for agent-specific Armington decompositions, though this increases the size of the model considerably. This is described further in Annex 3 on alternative trade specification.

$$GREV_{r,ptax} = \sum_a \left[\tau^p_{r,a} PX_{r,a} XP_{r,a} + \tau^x_{r,a} XP_{r,a}\right] \tag{Y-1}$$

$$GREV_{r,atax} = \sum_i \sum_{aa} \tau^{Ap}_{r,i,aa} (\gamma^c_{r,i,aa} PAT_{r,i}) XA_{r,i,aa} \tag{Y-2}$$

$$GREV_{r,ttax} = \sum_{i \in Arm} \sum_{r'} \tau^m_{r',r,i} WPM_{r',r,i} WTF^d_{r',r,i} + \sum_{i \notin Arm} \tau^m_{r,i} PW_i XMT_{r,i} \tag{Y-3}$$

$$GREV_{r,etax} = \sum_{i \in Arm} \sum_{r'} \tau^e_{r,r',i} PE_{r,r',i} WTF^s_{r,r',i} + \sum_{i \notin Arm} \tau^e_{r,i} PW_i XET_{r,i} \tag{Y-4}$$

$$GREV_{r,vtax} = \sum_{fp} \sum_a \tau^v_{r,fp,a} NPF_{r,fp,a} XF_{r,fp,a} \tag{Y-5}$$

$$GREV_{r,ctax} = \sum_{em} \sum_i \sum_{aa} \tau^{emi}_{r,em} \varphi_{r,em,i,aa} \chi^e_{em} \rho_{r,em,i,aa} XA_{r,i,aa} \tag{Y-6}$$

$$GREV_{r,htax} = \sum \chi^k_r \kappa^h_{r,h} YH_r \tag{Y-7}$$

Equation (Y-8) defines aggregate fiscal revenues, where the set *gy* corresponds to the six indirect tax streams (*ptax*, *atax*, *ttax*, *etax*, *vtax* and *ctax*) and the direct tax stream (*htax*). It is also assumed that income from a cap and trade system on emissions accrue to the government. Equation (Y-9) summarizes net household income, *YH*. It is assumed that all factor income net of factor taxes (where *NPF* represents the market clearing factor price net of taxes) accrues to households as well as profits generated by the markups. Household income is then adjusted for the depreciation allowance, *DeprY*. Equation (Y-10) describes household disposable income, *YD*, where κ^h represents the base year household-specific direct tax rate. The direct tax rate is adjusted by an economy-wide adjustment factor, χ^k, which can be endogenous to achieve a given target, for example the deficit of the public sector. Macro closure is discussed in more detail below.[9] Disposable income is adjusted by changes in international tourism receipts, *IIT*, which will be affected by climate change.

[9] The GTAP dataset contains only a single representative household per country/region. The model implementation allows for multiple households and hence the need for an economy-wide tax shifter that is uniform across households. This has different distributional consequences than an additive shifter or a more complex direct tax schedule.

(Y-8) $$YG_r = \sum_{gy} GREV_{r,gy} + \sum_{em} QuotaY_{em}^E$$

(Y-9) $$YH_r = \sum_{fp}\sum_{a} NPF_{r,fp,a} XF_{r,fp,a} + \sum_{a} \Pi_{r,a} - DeprY_r$$

(Y-10) $$YD_r = \left(1 - \chi_r^k \kappa_{r,h}^h\right) YH_r + IIT_{r,h}$$

Demand block

The demand block is divided into two sections. The first describes the allocation of household disposable income between savings and expenditures on goods and services. The second describes other final demand for goods and services.

Households first allocate total expenditures between savings on the one hand and aggregate expenditures on goods and services on the other hand.[10] Equation (D-1) determines the household savings rate (relative to disposable income), s^s, as a function of per capita growth (g^{pc}) and the youth and elderly dependency ratios, respectively given by $DRAT^{PLT15}$ and $DRAT^{P65UP}$.[11] These variables are typically exogenous in dynamic scenarios. The savings function also captures a persistence factor defined by β^s.[12] Equation (D-2) determines the level of household savings. It should be noted that if the ELES utility function is used to specify household demand for goods and services, equation (D-1) is dropped and equation (D-2) then defines the average propensity to save as the ELES itself determines the level of savings. Equation (D-3) in essence determines aggregate expenditures on household goods and services, *YC*. In the case of the ELES, combined with equation (D-4), it defines the level of household savings, which is an outcome of the ELES. Equation (D-4) is only used for the ELES version of the model.

10 The demand block is significantly reformulated compared to the first version of the ENVISAGE model. The latter was largely inspired by the GTAP model. The current version is more similar to the Linkage specification. It drops the top level utility function that allocated national income across savings, and public and private expenditures. In the long-term scenarios this top-level structure was typically over-ruled with other specific assumptions making the theoretical consistency of the top-level formulation less appealing.

11 The original theory and parameters for this formulation can be found in Loayza et al 2000 and Masson et al 1998 and is summarized in van der Mensbrugghe 2006.

12 Setting all the β parameters to 0 would yield a constant savings rate. It is also possible to use this equation to formulate a different closure—for example to target investment and allow the shift parameter, χ^s, to adjust to achieve the given target.

(D-1)	$s^s_{r,h} = \chi^s_r \alpha^s_{r,h} + \beta^s_r s^s_{r,h,-1} + \beta^g_r g^{pc}_r + \beta^y_r DRAT^{PLT15}_r + \beta^e_r DRAT^{P65UP}_r$
(D-2)	$S^h_{r,h} = s^s_{r,h} YD_{r,h}$
(D-3)	$YD_{r,h} = YC_{r,h} + S^h_{r,h}$
(D-4)	$YC_{r,h} = \sum_k PHX_{r,k,h} XH_{r,k,h}$

Expenditures

The next block of equations determines the sectoral demands for goods and services for households. In the standard model private expenditures are derived from the AIDADS specification and are based on consumer-defined goods, *XH*, indexed by *k*, not Armington goods, *XA*, that are indexed by *i*.[13] Equations (D-5) and (D-6) are used for three demand systems—LES, ELES and AIDADS. Equation (D-5) defines supernumerary income, Y^*, residual income after subtracting expenditures on subsistence minima. (In the case of the ELES, savings is added to *YC* as the savings decision is part of the allocation of disposable income and not determined independently. Thus δ^e equals 1 for the ELES implementation and 0 for all others.) Equation (D-6) determines household expenditure on good *k*. It is composed of two factors. The first factor represents expenditures on the subsistence minimum, θ^{gh}, or floor expenditures. These are calibrated on a per capita basis and are therefore multiplied by population to get the total volume. The second component is a share of supernumerary income, where μ is the marginal share and *PHX* represents the price of consumer commodity *k*. In the LES and ELES formulations, the marginal share parameters are calibrated and fixed. In the AIDADS formula, the marginal shares are a function of utility (intuitively of income), and thus the marginal shares evolve over time. Equation (D-7) determines the marginal shares, based on calibrated parameters α and β. Clearly, if the two are identical, we are back to an LES/ELES specification. Equation (D-8) defines the utility level, *U*—either explicitly or as an implicit function. The CDE implementation is described fully in Annex 2.

It should be noted that the subsistence minima, though calibrated using base year data, are in some cases 'climate sensitive'—for example energy demand for cooling and/or heating. This is described further below. The θ^{gh} parameters represent the climate sensitive parameters, whereas the θ^h are the initial calibrated parameters.

13 Other demand specifications have been implemented and are described in Annex 2. These include both the linear and extended linear expenditure system (LES and ELES) as well as the CDE specification that is the standard utility function for the GTAP model. Three of the expenditure systems (CDE, LES and AIDADS) use a two-tiered nest to allocate savings on the one hand and expenditures on goods and services on the other. The ELES integrates both in a single-tiered system.

(D-5) $$Y_{r,h}^{*} = YC_{r,h} + \delta^{e} S_{r,h}^{h} - Pop_{r,h} \sum_{k} \theta_{r,k,h}^{gh} PHX_{r,k,h}$$

(D-6) $$HX_{r,k,h} = Pop_{r,h} \theta_{r,k,h}^{gh} + \frac{\mu_{r,k,h}^{c}}{PHX_{r,k,h}} Y_{r,h}^{*}$$

(D-7) $$\mu_{r,k,h}^{c} = \frac{\alpha_{r,k,h}^{ad} + \beta_{r,k,h}^{ad} e^{U_{r,h}}}{1 + e^{U_{r,h}}}$$

(D-8) $$U_{r,h} = \sum_{k} \mu_{r,k,h}^{c} \ln\left(\frac{HX_{r,k,h}}{Pop_{r,h}} - \theta_{r,k,h}^{gh}\right) - \ln(A_{r,h}) - 1 \Leftrightarrow \sum_{k} \mu_{r,k,h}^{c} \ln\left(\frac{\dfrac{HX_{r,k,h}}{Pop_{r,h}} - \theta_{r,k,h}^{gh}}{A_{r,h} e^{U_{r,h}}}\right) \equiv 1$$

Equation (D-9) represents the private expenditure budget shares, s^h. Equation (D-10) defines a consumer price index for private consumption, *PC*. Finally, equation (D-11) defines the volume of aggregate private consumption, *XC*.

(D-9) $$s_{r,k,h}^{h} = \frac{PHX_{r,k,h} HX_{r,k,h}}{YC_{r,h}}$$

(D-10) $$PC_{r,h} = \sum_{k} s_{r,k,h}^{h} PHX_{r,k,h}$$

(D-11) $$XC_{r,h} = YC_{r,h} / PC_{r,h}$$

The next set of equations decomposes consumer demand defined as consumer goods into produced (or more accurately, Armington) goods. A transition matrix approach is used where each consumed good is composed of one or more produced goods and combined using a CES aggregator.[14] Each consumer good could also have its own energy bundle—with different demand shares across energy.[15] Equation (D-12) converts consumed goods *HX* into non-energy Armington goods, *XA*. Equation (D-13) determines demand for the energy bundle, *XNRG*, for each of the *k* consumed goods. There is a one-to-one correspondence between each index *k* and an index in the set *eb* (that also includes demand for energy bundles in production and other final demand). Equation (D-14) then determines the price of consumer good *k*.

14 Using the standard GTAP data, the transition matrix is diagonal—each consumed good corresponds to exactly one produced good. ENVISAGE still uses this approach save for the energy bundle that is combined into one consumed commodity. Work is ongoing to develop a global database of transition matrices. The GREEN model for example (see Burniaux et al 1992 and van der Mensbrugghe 1994) had four consumed goods and eight standard produced goods.

15 For example, a transportation bundle is likely to be dominated by liquid fuel demand, whereas demand for heat is likely to be dominated by electricity and natural gas.

$$XA_{r,n,h} = \sum_k \alpha^c_{r,n,k,h} \left(\frac{PHX_{r,k,h}}{PA^a_{r,n,h}} \right)^{\sigma^c_{r,k,h}} HX_{r,k,h} \tag{D-12}$$

$$XNRG_{r,eb,Old} = \sum_h \alpha^{nrgh}_{r,k,h} \left(\frac{PHX_{r,k,h}}{PNRG_{r,eb,Old}} \right)^{\sigma^c_{r,k,h}} HX_{r,k,h} \tag{D-13}$$

$$PHX_{r,k,h} = \left[\sum_{n \in nnrg} \alpha_{r,n,k,h} (PA^a_{r,n,h})^{1-\sigma^c_{r,k,h}} + \alpha^{nrgh}_{r,k,h} (PNRG_{r,eb,Old})^{1-\sigma^c_{r,k,h}} \right]^{1/(1-\sigma^c_{r,k,h})} \tag{D-14}$$

The final block of demand equations decomposes aggregate public and investment demands. A CES expenditure function is used that covers all non-energy Armington goods and an energy bundle. Decomposition of the energy bundle is done at a later stage. Equation (D-15) represents the sectoral (Armington) demand for public and investment non-energy expenditures *XA*, where the index *f* represents the set spanning (*gov* and *inv*). Equation (D-16) determines the demand for the energy bundle (where the index *eb* is mapped to the respective *f* index). The expenditure price indices, PC_f, are given by equation (D-17). In the standard model there are no stock-building activities. In some scenarios it is helpful to give 'exogenous' demand shocks. This is most easily done by assuming stock-building activities as defined in equation (D-18), where the level of stock-building is linked to domestic production, *XS*.

$$XA_{r,n,f} = \alpha^f_{r,n,f} \left(\frac{PC_{r,f}}{PA^a_{r,n,f}} \right)^{\sigma^f_{r,f}} XC_{r,f} \tag{D-15}$$

$$XNRG_{r,eb,Old} = \alpha^{nrgf}_{r,f} \left(\frac{PC_{r,f}}{XNRG_{r,eb,Old}} \right)^{\sigma^f_{r,f}} XC_{r,f} \tag{D-16}$$

$$PC_{r,f} = \left[\sum_n PA^a_{r,n,f} XA_{r,n,f} + PNRG_{r,eb,Old} XNRG_{r,eb,Old} \right] / XC_{r,f} \tag{D-17}$$

$$XA_{r,i,stb} = \chi^{stb}_{r,i} XS_{r,i} \tag{D-18}$$

The fuels block

Each agent in the economy has a specified demand for an aggregate energy bundle. The fuel demanders are indexed by *eb* that spans all activities (*a*), each commodity consumed by households (*k*) and other final demand (*f*).[16] The equations above provide the bundle *XNRG* across all *eb* agents. That bundle is decomposed across all energy sources using a nested CES

[16] With the simplified consumer transition matrix, only one consumed commodity demands energy, and that is the entire expenditure on energy in the single consumer demand vector in the existing GTAP data.

structure with agent-specific share parameters and substitution elasticities.[17] At the top level, demand is decomposed between electricity and non-electric energy (see figure 2). The non-electric bundle is split into coal on the one hand, and gas and oil on the other. The oil and gas bundle is then split into oil on the one hand and gas on the other. Using the standard GTAP classification, the final electric bundle is composed of commodity *ely* alone. The coal bundle is composed of the commodity *coa* alone. The gas bundle is composed of the commodities *gas* and *gdt*. And the oil bundle is composed of the commodities *oil* and *p_c*. In most cases, for these latter two bundles, one component will dominate the other. For example, there may be some residual *oil* consumption in households, but the bulk of the consumption will be *p_c*. When the new alternative technologies are introduced, they are inserted at the bottom most node for electricity, coal, oil and gas respectively.

The next block of equations is the top of the energy node nest. It decomposes the energy bundle, *XNRG*, into an electric bundle, *XELY*, and a non-electric bundle, *XNELY*. Equations (F-1) and (F-2) define respectively the demands for the electric and non-electric bundles with a substitution elasticity given by σ^e. The equations are defined over all demanders of energy bundles, *eb*, and are also vintage-specific in production. Equation (F-3) defines the CES price of the energy bundle as a CES aggregation of the respective bundle prices, *PELY* and *PNELY*.

(F-1) $$XELY_{r,eb,v} = \alpha^{ely}_{r,eb,v}\left(\frac{PNRG_{r,eb,v}}{PELY_{r,eb,v}}\right)^{\sigma^e_{r,eb,v}} XNRG_{r,eb,v}$$

(F-2) $$XNELY_{r,eb,v} = \alpha^{nely}_{r,eb,v}\left(\frac{PNRG_{r,eb,v}}{PNELY_{r,eb,v}}\right)^{\sigma^e_{r,eb,v}} XNRG_{r,eb,v}$$

(F-3) $$PNRG_{r,eb,v} = \left[\alpha^{ely}_{r,eb,v}\left(PELY_{r,eb,v}\right)^{1-\sigma^e_{r,eb,v}} + \alpha^{nely}_{r,eb,v}\left(PNELY_{r,eb,v}\right)^{1-\sigma^e_{r,eb,v}}\right]^{1/(1-\sigma^e_{r,eb,v})}$$

The following block decomposes the non-electric bundle into a coal bundle, *XCOA*, and an oil and gas bundle, *XOLG*, given respectively by equations (F-4) and (F-5) with a substitution elasticity of σ^{nely}. Equation (F-6) defines the price of the non-electric bundle, P^{NELY}.

[17] Note that in the case of energy bundles from the production side, they are also indexed by vintage with potentially different substitution elasticities across vintages. The non-production energy bundles are also indexed by vintage (for simplicity), though only the *Old* vintage is active.

$$XCOA_{r,eb,v} = \alpha^{coa}_{r,eb,v}\left(\frac{PNELY_{r,eb,v}}{PCOA_{r,eb,v}}\right)^{\sigma^{nely}_{r,eb,v}} XNELY_{r,eb,v} \tag{F-4}$$

$$XOLG_{r,eb,v} = \alpha^{olg}_{r,eb,v}\left(\frac{PNELY_{r,eb,v}}{POLG_{r,eb,v}}\right)^{\sigma^{nely}_{r,eb,v}} XNELY_{r,eb,v} \tag{F-5}$$

$$PNELY_{r,eb,v} = \left[\alpha^{coa}_{r,eb,v}\left(PCOA_{r,eb,v}\right)^{1-\sigma^{nely}_{r,eb,v}} + \alpha^{olg}_{r,eb,v}\left(POLG_{r,eb,v}\right)^{1-\sigma^{nely}_{r,eb,v}}\right]^{1/(1-\sigma^{nely}_{r,eb,v})} \tag{F-6}$$

The third node decomposes the oil and gas bundle into a gas bundle, *XGAS*, and an oil bundle, *XOIL*. Equations (F-7) and (F-8) provide the demand equations for the respective bundles with a substitution elasticity of σ^{olg}. Finally, equation (F-9) describes the price of the oil and gas bundle, *POLG*, as a CES aggregation of the gas bundle, *PGAS*, and the oil bundle, *POIL*.

$$XGAS_{r,eb,v} = \alpha^{gas}_{r,eb,v}\left(\frac{POLG_{r,eb,v}}{PGAS_{r,eb,v}}\right)^{\sigma^{olg}_{r,eb,v}} XOLG_{r,eb,v} \tag{F-7}$$

$$XOIL_{r,eb,v} = \alpha^{oil}_{r,eb,v}\left(\frac{POLG_{r,eb,v}}{POIL_{r,eb,v}}\right)^{\sigma^{olg}_{r,eb,v}} XOLG_{r,eb,v} \tag{F-8}$$

$$POLG_{r,eb,v} = \left[\alpha^{gas}_{r,eb,v}\left(PGAS_{r,eb,v}\right)^{1-\sigma^{olg}_{r,eb,v}} + \alpha^{oil}_{r,eb,v}\left(POIL_{r,eb,v}\right)^{1-\sigma^{olg}_{r,eb,v}}\right]^{1/(1-\sigma^{olg}_{r,eb,v})} \tag{F-9}$$

At this point, the decomposition of fuels is down to four fundamental energy sources—electricity, coal, gas and oil. In the initial state, with the GTAP data alone, each of the six energies in GTAP is mapped to these four bundles. Four energy sets are defined: *ely*, *coa*, *oil* and *gas* that correspond to a mapping to one of the four types of energy. The GTAP *ely* sector is mapped to *ely*, the GTAP *coa* sector is mapped to *coa*, the GTAP *gas* and *gdt* sectors are mapped to *gas*, and the GTAP *oil* and *p_c* sectors are mapped to *oil*. With the introduction of new technologies, the set mappings will increase. Thus if there is one electric backstop technology, say renewables, and designated by *elybs*, it will be mapped to the *ely* aggregate electric bundle.

$$XA_{r,ely,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{elybs}_{r,ely,eb,v} \left(\lambda^{e}_{r,ely,eb,v}\right)^{\sigma^{ely}_{r,eb,v}-1} \left(\frac{PELY_{r,eb,v}}{PA^{a}_{r,ely,aa}}\right)^{\sigma^{ely}_{r,eb,v}} XELY_{r,eb,v} \tag{F-10}$$

$$PELY_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{ely} \alpha^{elybs}_{r,ely,eb,v} \left(\frac{PA^{a}_{r,ely,aa}}{\lambda^{e}_{r,ely,eb,v}}\right)^{1-\sigma^{ely}_{r,eb,v}}\right]^{1/(1-\sigma^{ely}_{r,eb,v})} \tag{F-11}$$

$$XA_{r,coa,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{coabs}_{r,coa,eb,v} \left(\lambda^{e}_{r,coa,eb,v}\right)^{\sigma^{coa}_{r,eb,v}-1} \left(\frac{PCOA_{r,eb,v}}{PA^{a}_{r,coa,aa}}\right)^{\sigma^{coa}_{r,eb,v}} XCOA_{r,eb,v} \tag{F-12}$$

$$PCOA_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{coa} \alpha^{coabs}_{r,coa,eb,v} \left(\frac{PA^{a}_{r,coa,aa}}{\lambda^{e}_{r,coa,eb,v}}\right)^{1-\sigma^{coa}_{r,eb,v}}\right]^{1/(1-\sigma^{coa}_{r,eb,v})} \tag{F-13}$$

$$XA_{r,gas,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{gasbs}_{r,gas,eb,v} \left(\lambda^{e}_{r,gas,eb,v}\right)^{\sigma^{gas}_{r,eb,v}-1} \left(\frac{PGAS_{r,eb,v}}{PA^{a}_{r,gas,aa}}\right)^{\sigma^{gas}_{r,eb,v}} XGAS_{r,eb,v} \tag{F-14}$$

$$PGAS_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{gas} \alpha^{gasbs}_{r,gas,eb,v} \left(\frac{PA^{a}_{r,gas,aa}}{\lambda^{e}_{r,gas,eb,v}}\right)^{1-\sigma^{gas}_{r,eb,v}}\right]^{1/(1-\sigma^{gas}_{r,eb,v})} \tag{F-15}$$

$$XA_{r,oil,aa} = \sum_{eb \in aa} \sum_{v} \alpha^{oilbs}_{r,oil,eb,v} \left(\lambda^{e}_{r,oil,eb,v}\right)^{\sigma^{oil}_{r,eb,v}-1} \left(\frac{POIL_{r,eb,v}}{PA^{a}_{r,oil,aa}}\right)^{\sigma^{oil}_{r,eb,v}} XOIL_{r,eb,v} \tag{F-16}$$

$$POIL_{r,eb,v} = \left[\sum_{aa \in eb} \sum_{oil} \alpha^{oilbs}_{r,oil,eb,v} \left(\frac{PA^{a}_{r,oil,aa}}{\lambda^{e}_{r,oil,eb,v}}\right)^{1-\sigma^{oil}_{r,eb,v}}\right]^{1/(1-\sigma^{oil}_{r,eb,v})} \tag{F-17}$$

Equations (F-10) through (F-17) determine the decomposition of the four basic energy bundles to their respective Armington volumes. For electricity and coal, with the base data, these equations are somewhat redundant since the bundles map to only one Armington commodity. Each demand equation requires a summing over vintages (for only activities), and a summing across *eb* indices. In most cases, the *eb* index maps to one, and only one, agent (*aa*). In the case of consumption, however, the energy bundle can exist for each consumed commodity (k), and thus there can be as many energy decompositions as there are consumer commodities. Each bundle also allows for energy efficiency improvement, sometimes designated as the autonomous energy efficiency improvement (AEEI) parameter, which is region, agent, fuel and vintage specific (in principle). The price equations need a separate mapping from the *eb* to the *aa* index, though it is assumed that the consumer price for a given fuel is uniform across the *k* commodities (i.e. natural gas used for heat has the same price as natural gas used for transportation.).

Trade block

Top level Armington

The equations above have determined completely the so-called Armington demand for goods across all agents, *XA*, that include activities (*a*), private or consumer demand (*h*), and other final demand (*f*). The union of these three sets is the set *aa*. In the standard version of ENVISAGE, all Armington agents are assumed to have the same preference function for domestic and import goods.[18] It is also assumed that the Armington good, for each commodity *i*, is homogeneous across agents, and can therefore be aggregated in volume terms. However, when using the energy volume data that comes with the GTAP data set, the derived energy prices vary (modestly in most cases) across agents.[19] To maintain the adding up assumption with the price differentials, a shift parameter is associated with each agent. One could think of this intuitively as a quality index, so the gasoline consumed by households has a different quality than that consumed in transportation, where quality differences may simply reflect octane levels.

Equation (T-1) defines aggregate Armington demand, *XAT*. It is the sum across all agents of their Armington demand—adjusted by the fixed shift (or quality) parameter, γ^c.[20] The agent-specific Armington price is composed of two components. The first, *PA*, is formed from the nationally determined Armington price, *PAT*, defined below, adjusted by the quality index, γ^c, and augmented by the user-specific sales tax, τ^{Ap}—see equation (T-2). To this is added the emission tax, τ^{emi}, see equation (T-3). The emissions tax is given as a $ amount per unit of emission, where ρ determines the agent specific level of emissions per unit of demand by agent (*aa*), per input (*i*) and per emission type (*em*). The emissions rate ρ is multiplied by a global emissions factor χ^e that allows for the emissions rate to vary in the baseline scenario to achieve a given global emissions trend.[21] In other words ρ is calibrated to base year data and χ^e represents trend changes in the emissions rate. The model allows for full or partial exemptions using the parameter ϕ—that can also be agent, input and emission specific. For example it is possible to exempt given sectors or households from paying the emissions tax for specific fuels, say gasoline. By default, the parameter ϕ is set at 1, i.e. there are no exemptions. The emissions tax can be either set exogenously or be model-derived by imposing an emissions cap at either the country or regional level.[22] Notice that the emission tax is not an ad valorem tax, but a Pigouvian per unit tax.

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18 The GTAP data decomposes Armington demand into its domestic and import component by agent. Annex 3 explains an alternative version of the Armington decomposition that allows for agent-specific behavior. Note that this increases the size of the model considerably.

19 The energy data, derived from the databases of the International Energy Agency (IEA), are expressed in millions of tons of oil equivalent (MTOE) across all energies, and thus prices are $2004 prices per unit of MTOE.

20 The γ^c parameter is initialized at 1 for all non-energy commodities. For energy commodities, it is initialized such that there is uniformity of energy prices in efficiency units.

21 The baseline calibration of the emission rates only affects non-CO_2 greenhouse gases.

22 The model does not include equation (T-3) as it has been substituted throughout to minimize the creation of additional variables.

(T-1) $$XAT_{r,i} = \sum_{aa} \gamma^c_{r,i,aa} XA_{r,i,a}$$

(T-2) $$PA_{r,i,aa} = (1+\tau^{Ap}_{r,i,aa})\gamma^c_{r,i,aa} PAT_{r,i}$$

(T-3) $$PA^a_{r,i,aa} = PA_{r,i,aa} + \sum_{em} \tau^{emi}_{r,em} \chi^e_{em} \varphi_{r,em,i,aa} \rho_{r,em,i,aa}$$

As described above, the decomposition of the Armington aggregate, *XAT*, is done at the national level. (The Armington equations are all indexed by *im*. The model allows for homogeneous traded commodities and these are indexed by *ih*.) Aggregate national demand for domestic goods, *XD*, is then a fraction of *XAT*, with the fraction sensitive to the relative price of domestic goods, *PD*, to the Armington good, *PAT*—as shown in equation (T-4). The key parameter, known as the Armington substitution elasticity, is σ^m. The model allows for quality differences in the Armington composite goods using the γ^a and γ^t parameters. These in effect allow one to calibrate the CES functions in terms of value shares with the appropriate initialization of the respective γ parameters. Equation (T-5) determines the demand for aggregate imports, *XMT*, which are further decomposed by trading partner (see below). The price of aggregate imports is tariff-inclusive. Finally, equation (T-6) defines the aggregate (or national) price of the aggregate Armington good, *PAT*.

(T-4) $$XD_{r,im} = \alpha^d_{r,im} \left(\gamma^a_{r,im}\right)^{\sigma^m_{r,im}-1} \left(\frac{PAT_{r,im}}{PD_{r,im}}\right)^{\sigma^m_{r,im}} XAT_{r,i,a}$$

(T-5) $$XMT_{r,im} = \alpha^m_{r,im} \left(\gamma^t_{r,im}\right)^{\sigma^m_{r,im}-1} \left(\frac{PAT_{r,im}}{PMT_{r,im}}\right)^{\sigma^m_{r,im}} XAT_{r,i,a}$$

(T-6) $$PAT_{r,im} = \left[\alpha^d_{r,im}\left(\frac{PD_{r,im}}{\gamma^a_{r,im}}\right)^{1-\sigma^m_{r,im}} + \alpha^m_{r,im}\left(\frac{PMT_{r,im}}{\gamma^t_{r,im}}\right)^{1-\sigma^m_{r,i,a}}\right]^{1/(1-\sigma^m_{r,i,a})}$$

Each bilateral trade flow is associated with four different prices:

1. *PE* represents the factory or farm gate price
2. *WPE* represents the FOB price, an export tax or subsidy induces a wedge between the producer price and the FOB price[23]
3. *WPM* represents the CIF price, international trade and transport margins introduce a wedge between the FOB and CIF price
4. *PM* represents the agent-price and includes the bilateral tariff

[23] The ENVISAGE model specification of export taxes is that they are an ad valorem tax on the producer price, thus an export subsidy is negative. An alternative formulation would be to specify the tax as a wedge between the world price and the domestic FOB price in which case the subsidy is measured as a positive wedge.

Equations (T-7) through (T-9) describe three of the prices associated with international trade, respectively *WPE*, *WPM* and *PM* (the determination of *PE* is described below). The respective wedges are represented by τ^e, the export tax/subsidy, τ^{tm}, the international transport margin, and τ^m the bilateral tariff. The price of a unit of international transport is uniform, irrespective of the transport node and sector.

Second level Armington nest

The second nest in the Armington structure allocates aggregate import demand (across all agents) to specific regions of origin.[24] The bilateral trade flow will reflect preferences, the region of origin-specific export price and the bilateral tariff, τ^m. The price impacts are reflected in the tariff-inclusive bilateral price *PM*. Equation (T-10) defines import demand, WTF^d, by region *r*, sourced in region *r*'. Equation (T-11) defines the aggregate import price, *PMT*. It is an aggregation of the tariff inclusive bilateral import price. All agents are assumed to face the same import price (net of the sales tax), i.e. implicitly we are assuming that the composition of the import bundle by each agent is identical.

(T-7) $$WPE_{r,r',im} = (1+\tau^e_{r,r',im})PE_{r,r',im}$$

(T-8) $$WPM_{r,r',im} = WPE_{r,r',im} + \tau^{tm}_{r,r',im}PWMG$$

(T-9) $$PM_{r,r',im} = (1+\tau^m_{r,r',im})WPM_{r,r',im}$$

(T-10) $$WTF^d_{r',r,im} = \alpha^w_{r',r,im}\left(\gamma^m_{r'r,im}\right)^{\sigma^w_{r,im}-1}\left(\frac{PMT_{r,im}}{PM_{r',r,im}}\right)^{\sigma^w_{r,im}} XMT_{r,im}$$

(T-11) $$PMT_{r,im} = \left[\sum_{r'}\alpha^w_{r',r,im}\left(\frac{PM_{r',r,im}}{\gamma^m_{r',r,im}}\right)^{1-\sigma^w_{r,im}}\right]^{1/(1-\sigma^w_{r,im})}$$

Export supply

Analogous to the two-nested Armington specification described above, the ENVISAGE model allows for imperfect transformation of output across markets of destination—domestic and for export. A two-nested CET structure is implemented. At the top level, output is allocated between the domestic market and aggregate exports. At the next level, aggregate exports are allocated across various foreign markets. At either nest, infinite transformation is allowed in which case the CET first order conditions are replaced by the law of one price. The supply of international trade and transport services (*XMG*) is treated apart and is assumed to be priced at the average producer price, *PP*.

[24] Note that in either version of the top-level Armington decomposition—national or agent-specific—the decomposition of imports by region of origin is specified at the national level.

Equations (T-12) and (T-13) represent the derived supply for domestic, *XDT*, and aggregate export, *XET*, markets respectively. With finite transformation, these conditions are the standard CET first order conditions based on supply (less supply of international trade and transport services). With perfect transformation, each is replaced with the law of one price whereby the domestic, *PD*, and export, *PET*, producer price are set equal to the aggregate supply price, *PS*. Equation (T-14) represents the market equilibrium for supply. With perfect transformation domestic supply is equal to the sum of supply to the various markets—domestic, *XDT*, aggregate exports, *XET*, and international trade and transport services, *XMG*. With finite transformation, the aggregation function is equal to the CET primal function. However, this can be replaced with the CET dual price function as is the case in equation (T-14). All equations allow for a component specific quality or efficiency factor, γ^d and γ^e.

(T-12)
$$\begin{cases} PD_{r,im} = \gamma^d_{r,im} PS_{r,im} & \text{if} \quad \sigma^x_{r,im} = \infty \\ XDT^s_{r,im} = \gamma^{xd}_{r,im} \left(\gamma^d_{r,im}\right)^{\sigma^x_{r,im}-1} \left(\dfrac{PD_{r,im}}{PS_{r,im}}\right)^{\sigma^x_{r,im}} \left[XS_{r,im} - XMG_{r,im}\right] & \text{if} \quad \sigma^x_{r,im} < \infty \end{cases}$$

(T-13)
$$\begin{cases} PET_{r,im} = \gamma^e_{r,im} PS_{r,im} & \text{if} \quad \sigma^x_{r,im} = \infty \\ XET_{r,im} = \gamma^{xe}_{r,im} \left(\gamma^e_{r,im}\right)^{\sigma^x_{r,im}-1} \left(\dfrac{PET_{r,im}}{PS_{r,im}}\right)^{\sigma^x_{r,im}} \left[XS_{r,im} - XMG_{r,im}\right] & \text{if} \quad \sigma^x_{r,im} < \infty \end{cases}$$

(T-14)
$$\begin{cases} XS_{r,im} = \gamma^d_{r,im} XDT^s_{r,im} + \gamma^e_{r,im} XET_{r,im} + XMG_{r,im} & \text{if} \quad \sigma^x_{r,i} = \infty \\ PS_{r,i} = \left[\gamma^{xd}_{r,im} \left(\dfrac{PD_{r,im}}{\gamma^d_{r,im}}\right)^{1+\sigma^x_{r,im}} + \gamma^{xe}_{r,im} \left(\dfrac{PET_{r,im}}{\gamma^e_{r,im}}\right)^{1+\sigma^x_{r,im}}\right]^{1/(1+\sigma^x_{r,im})} & \text{if} \quad \sigma^x_{r,i} < \infty \end{cases}$$

Equations (T-15) and (T-16) reflect the second level CET nest allocating aggregate exports, *XET*, across various export markets as represented by WTF^s. With perfect transformation, the bilateral export producer price is equal to the aggregate export price, *PET*, and aggregate export supply is simply the sum across all export markets. With finite transformation, the CET first-order condition determines WTF^s and the aggregate export price is the CET aggregation of the regional export prices. Similar to the other trade equations, a quality or efficiency parameter is introduced that allows for prices to deviate from uniformity even with an infinite transformation elasticity.

$$\text{(T-15)} \quad \begin{cases} PE_{r,r',im} = \gamma^{w}_{r,r',im} PET_{r,im} & \text{if} \quad \sigma^{z}_{r,im} = \infty \\ WTF^{s}_{r,r',im} = \gamma^{xw}_{r,r',im} \left(\gamma^{w}_{r,r',im}\right)^{\sigma^{z}_{r,im}-1} \left(\dfrac{PE_{r,r',im}}{PET_{r,im}}\right)^{\sigma^{z}_{r,im}} XET_{r,im} & \text{if} \quad \sigma^{z}_{r,im} < \infty \end{cases}$$

$$\text{(T-16)} \quad \begin{cases} XET_{r,im} = \sum_{i} \gamma^{w}_{r,r',im} WTF^{s}_{r,r',im} & \text{if} \quad \sigma^{z}_{r,im} = \infty \\ PET_{r,im} = \left[\sum_{r'} \gamma^{w}_{r,r',im} \left(\dfrac{PE_{r,r',im}}{\gamma^{w}_{r,r',im}}\right)^{1+\sigma^{z}_{r,im}}\right]^{1/(1+\sigma^{z}_{r,im})} & \text{if} \quad \sigma^{z}_{r,im} < \infty \end{cases}$$

Homogeneous traded goods

The model allows for homogeneous traded goods. In principle, none of the goods in GTAP can be treated immediately as homogeneous goods since there exists bilateral trade for all goods. In principle, some goods are nearly homogeneous since either imports or exports are so small that they could be ignored in an intermediate step that moves from the Armington specification to one based on net trade. It is also possible to introduce new commodities into the model as either Armington or homogeneous goods.

Equation (T-17) defines net trade, *NT*, for homogeneous goods defined over index *ih*. It is defined as a value and is the difference between domestic supply, *XS*, and domestic demand, *XAT*, evaluated at the world price, *PW*. Net trade is negative if demand exceeds supply. Equation (T-18) is the market equilibrium condition for homogeneous goods. At equilibrium, the sum of net trade across all countries must equal 0. Equation (T-19) determines the domestic price of homogeneous goods—it is equal to the world price adjusted by the tariff (that is no longer region of origin specific). Both supply and demand prices are equal as provided by equation (T-20).

$$\text{(T-17)} \quad NT_{r,ih} = PW_{ih}(XS_{r,ih} - XAT_{r,ih})$$

$$\text{(T-18)} \quad \sum_{r} NT_{r,ih} = 0$$

$$\text{(T-19)} \quad PS_{r,ih} = (1+\tau^{m}_{r,ih})PW_{ih}$$

$$\text{(T-20)} \quad PAT_{r,ih} = PS_{r,ih}$$

The next three equations are not strictly necessary for the model, but provide identities that can be useful. The first, (T-21), defines the volume of aggregate exports. It is specified as a mixed complementarity formula, or using an orthogonality condition. For the relation to hold, exports must be equal to net trade, or if net trade is negative, exports are set to zero, i.e. they must never fall below zero. The second equation (T-22), almost identical, defines the aggregate volume of imports. If exports are positive, *XMT* will be zero, else, exports are equal to the negative of net trade and will be positive. The third is a definition of a world price for Armington goods, and is a weighted global average of domestic supply prices, *PS*.

(T-21)	$(PW_{ih} XET_{r,ih} - NT_{r,ih}).XET_{r,ih} = 0$	and $XET_{r,ih} \geq 0$
(T-22)	$NT_{r,ih} = PW_{ih}(XET_{r,ih} - XMT_{r,ih})$	
(T-23)	$PW_{im,t} = \sum_r \varphi^w_{r,im,t} PS_{r,im,t}$	

Domestic supply

The model allows for multi-output production activities (for example producing ethanol and DDGS from ethanol production) and the aggregation of goods produced by activities into a single commodity (for example different streams of electrical production—coal, gas, hydro, nuclear, renewables, etc.—each with their own cost structure, but combined by a distributor into a single commodity).

Activity *a* can therefore produce a suite of commodities indexed by *i*, hence an output at this level is indexed by both *a* and *i*, $X_{a,i}$.[25] This is implemented using a CET structure with the possibility of infinite transformation. Equation (T-24) defines the supply of $X_{a,i}$ emanating from activity *a* (or XP_a), where the law of one price holds in the case of a finite transformation. Equation (T-25) represents the zero profit condition, or the revenue balance for the multi-output production function.

(T-24)
$$\begin{cases} X_{r,a,i} = \gamma^p_{r,a,i} \left(\dfrac{P_{r,a,i}}{PP_{r,a}} \right)^{\omega^s_{r,a}} XP_{r,i} & \text{if} \quad \omega^s_{r,a} < \infty \\ P_{r,a,i} = PP_{r,a} & \text{if} \quad \omega^s_{r,a} = \infty \end{cases}$$

(T-25)
$$PP_{r,a} XA_{r,a} = \sum_{i \in \{\gamma^p_{r,a,i} \neq 0\}} P_{r,a,i} X_{r,a,i}$$

In the next step, multiple streams of output can be combined into a single supplied commodity, XS_i, with a CES-aggregator. The specification allows for homogeneous goods, for example electricity—in which case the cost of each component must be equal, subject perhaps to an efficiency differential. Equation (T-26) determines the demand for produced commodity *X*. In the case of a finite elasticity it is a CES formulation. With an infinite substitution elasticity, the law-of-one price must hold, i.e. the producer price of each component must be equalized in

25 In the GTAP database this will be represented by a diagonal matrix where each activity produces one and only one good.

efficiency units. Equation (T-27) determines the equilibrium condition in the form of the cost function equality.

$$\text{(T-26)} \quad \begin{cases} X_{r,a,i} = \alpha^s_{r,a,i}(\gamma^s_{r,a})^{\sigma^s_{r,i}-1}\left(\dfrac{PS_{r,i}}{P_{r,a,i}}\right)^{\sigma^s_{r,i}} XS_{r,i} & \text{if} \quad \sigma^s_{r,i} < \infty \\ P_{r,a,i} = \gamma^s_{r,a} PS_{r,i} & \text{if} \quad \sigma^s_{r,i} = \infty \end{cases}$$

$$\text{(T-27)} \quad PS_{r,i} XS_{r,i} = \sum_{a \in \{\alpha^s_{r,a,i} \neq 0\}} P_{r,a,i} X_{r,a,i}$$

International trade and transport services

The global demand for international trade and transport services will be driven by the overall level of trade. Its allocation across suppliers is specified as a CES function where demand (partially) adjusts to low-cost suppliers. Within each region, production of these services is given by a CES technology.

Equation (T-28) determines the global demand for international trade and transport services, *XWMG*.[26] Regional supply of these services, *XTMG*, is determined in equation (T-29), the CES first order conditions. The global price, *PWMG*, is given in equation (T-30), the CES dual price formula. The regional supply price, *PTMG*, is given in equation (T-31). And the sectoral and regional supply of these services, *XMG*, is given in equation (T-32).

$$\text{(T-28)} \quad PWMG.XWMG = \sum_r \sum_{r'} \sum_{im} \left(WPM_{r,r',im} - WPE_{r,r',im}\right) WTF^s_{r,r',im}$$

$$\text{(T-29)} \quad XTMG_r = \alpha^{tmg}_r \left(\frac{PWMG}{PTMG_r}\right)^{\sigma^t} XWMG$$

$$\text{(T-30)} \quad PWMG = \left[\sum_r \alpha^{tmg}_r PTMG_r^{1-\sigma^t}\right]^{1/(1-\sigma^t)}$$

$$\text{(T-31)} \quad PTMG_r = \left[\sum_i \alpha^{mg}_{r,i} PP_{r,i}^{1-\sigma^{rt}_r}\right]^{1/(1-\sigma^{rt}_r)}$$

$$\text{(T-32)} \quad XMG_{r,i} = \alpha^{mg}_{r,i} \left(\frac{PTMG_r}{PP_{r,i}}\right)^{\sigma^{rt}_r} XTMG_r$$

[26] Note that the current formulation assumes that homogeneous goods are transported at no cost internationally.

Product market equilibrium

The model has only two 'basic' commodities—domestically produced goods for the domestic market, *XDT*, and bilateral exports, *WTF*. All other goods are composite goods. Equations (E-1) and (E-2) determine the equilibrium price for these two sets of goods, respectively *PD* and *PE*. With perfect transformation (at both levels), the true goods market equilibrium price is *PS* and equation (T-14) is the market equilibrium condition. In the model implementation, the equilibrium conditions (E-1) and (E-2) are substituted out.

(E-1) $$XDT^d_{r,i} = XDT^s_{r,i}$$

(E-2) $$WTF^d_{r,r',i} = WTF^s_{r,r',i}$$

Factor market equilibrium

The GTAP database has five factors of production—unskilled and skilled labor, capital, land and natural resources (or sector-specific factors: forestry, fishing, coal, oil, natural gas and other mining).[27] The next sections describe factor market equilibrium for these factors. The first describes a resource with a national market—with no, partial or full mobility. In the standard version of ENVISAGE, this covers only the aggregate land market.[28] Labor markets are covered separately. The model allows for labor market segmentation where the rural and urban markets clear separately and with the existence of a Harris-Todaro type rural to urban migration function. Natural resources have a supply curve under various assumptions. Finally, the capital market is handled apart—partially to implement the vintage capital structure.

Economy-wide factor markets

In the standard version of ENVISAGE land markets are national, i.e. economy-wide markets ranging from no mobility to full mobility. In the comparative static model, capital markets are treated the same way, but the dynamic version of the model, with vintage capital, has a somewhat different structure.

Clearance on national markets is governed by the degree of mobility across sectors and is modeled using a constant-elasticity-of-transformation specification. With an infinite transformation elasticity, factors of production are perfectly mobile across sectors and the law of one price holds. With finite (and even zero) transformation elasticity, factors are only partially mobile (or sector-specific) and factor returns are sector specific.

Equation (F-1) first determines aggregate national supply, *XFT*. The index *fpn* covers all nationally allocated factors of production—by default just land, and capital in the comparative static version of the model. There are two specification—either a constant elasticity specification

[27] GTAP also includes an additional satellite account that divides the land resource into 18 agro-ecological land types (AEZs). These have not been integrated in the standard version of ENVISAGE but have been implemented in a specialized version focused on bio-fuels (see Beghin et al 2010 [to be checked]).

[28] In the comparative static version of the model, it also is used for capital allocation.

or a logistic function. For land, typically the logistic function will be used so that total land supply never surpasses a maximum limit (currently calibrated to FAO data). Equation (F-2) then determines the sectoral supply allocation using a CET formulation with a finite elasticity for partial mobility or else with the imposition of the law of one price with perfect mobility. The law of one price holds, and thus the sectoral (net of tax) return, *NPF*, is equal to the economy-wide return, *PFT*. Equation (F-3) is then the aggregation condition. With perfect mobility it simply equates aggregate demand to aggregate supply. With partial mobility, it is replaced by the CET dual price formula for the aggregate or average price of land. Equation (F-4) is the market equilibrium condition equating sectoral supply to sectoral demand. With finite transformation it is a true market equilibrium condition, with perfect mobility it trivially sets sectoral supply equal to sectoral demand and equation (F-2) becomes the market equilibrium condition. Note that in the GAMS implementation, equation (F-4) is substituted out.

(F-1)
$$\begin{cases} XFT_{r,fpn} = \alpha^{ft}_{r,fpn}\left(\dfrac{PFT_{r,fpn}}{PGDPMP_r}\right)^{\varepsilon^f_{r,fpn}} & \text{if} \quad XFT^{max}_{r,fpn} = \infty \\ XFT_{r,fpn} = \dfrac{XFT^{max}_{r,fpn}}{1+\alpha^{ft}_{r,fpn}e^{-\gamma^{fts}_{r,fpn}(PFT_{r,fpn}/PGDPMP_r)}} & \text{if} \quad XFT^{max}_{r,fpn} < \infty \end{cases}$$

(F-2)
$$\begin{cases} NPF_{r,fpn,a} = PFT_{r,fpn} & \text{if} \quad \omega^f_{r,fpn} = \infty \\ XF^s_{r,fpn,a} = \alpha^{fs}_{r,fpn,a}\left(\dfrac{NPF_{r,fpn,a}}{PFT_{r,fpn}}\right)^{\omega^f_{r,fp}} XFT_{r,fpn} & \text{if} \quad \omega^f_{r,fpn} < \infty \end{cases}$$

(F-3)
$$\begin{cases} XFT_{r,fpn} = \sum_a XF^d_{r,fpn,a} & \text{if} \quad \omega^f_{r,fp} = \infty \\ PFT_{r,fpn} = \left[\sum_a \alpha^{fs}_{r,fpn,a} NPF^{1+\omega^f_{r,fp}}_{r,fpn,a}\right]^{1/(1+\omega^f_{r,fp})} & \text{if} \quad \omega^f_{r,fp} < \infty \end{cases}$$

(F-4)
$$XF^s_{r,fpn,a} = XF^d_{r,fpn,a}$$

Labor markets

In the standard ENVISAGE model, labor markets clear nationally with an economy-wide wage rate equating supply and aggregate demand—separately for both skilled and unskilled labor. The model does not allow for international migration. An alternative version of the model allows for national labor market segmentation with a Harris-Todaro type migration function from rural to urban activities. Due to data limitations, rural activities are equated with agricultural sectors and urban activities with all other sectors.

Sectoral labor demand across sectors (indexed by *a*) is determined by the production function in each sector. Sectors are segmented into two 'zones'—rural and urban, indexed by *z*. The basic idea behind Harris-Todaro is that migration is a function of the ratio of the urban wage to the rural wage. Equation (F-5) defines the average wage in each zone z, W^a. It is equal to total nominal labor remuneration in each zone divided by total volume demand (in person-years for example). Equation (F-6) then determines the level of migration from rural to urban zones, *MG*, as a function of the ratio of the nominal average wage in each zone (potentially adjusted for unemployment, i.e. the expected average wage), subject to a migration elasticity (ω^m), where χ^m is a calibrated shift parameter. Equation (F-7) then determines the zone-specific labor supply, L^s. It is equated to the previous period's labor supply adjusted by a zone-specific (and exogenous) labor supply growth rate and adjusted for migration. The parameter δ^z is equal to -1 for the rural zone and equal to +1 for the urban zone. In the case of no labor market segmentation, *MG* is equal to zero.[29] Equation (F-8) represents the equilibrium condition for the two possible specifications. The top equation equates supply by zone to demand by zone (under the assumption of full employment) with segmented markets. The bottom equation holds for the case with a nationally integrated labor market. Finally, equation (F-9) sets the sectoral wage. With segmented markets it is equal to the equilibrium wage in the relevant zone—potentially adjusted by a sector-specific wage premium that allows for inter-sectoral wage differences. With national markets, it is equal to the national equilibrium wage rate with again the possibility of a wage premium.

(F-5) $$W^a_{r,l,z} = \frac{\sum_{a \in z} NPF_{r,l,a} XF_{r,l,a}}{\sum_{a \in z} XF_{r,l,a}}$$

(F-6) $$MG_{r,l} = \chi^m_{r,l} \left(\frac{(1-UE_{r,l,Urb})W^a_{r,l,Urb}}{(1-UE_{r,l,Rur})W^a_{r,l,Rur}} \right)^{\omega^m_{r,l}} \quad \text{if} \quad \omega^m_{r,l} < \infty$$

(F-7) $$L^s_{r,l,z,t} = \left(1+g^l_{r,l,z,t}\right) L^s_{r,l,z,t-1} + \delta_z MG_{r,l,t}$$

(F-8) $$\begin{cases} L^s_{r,l,z} = \sum_{a \in z} XF_{r,l,a} & \text{if} \quad \omega^m_{r,l} < \infty \\ \sum_z L^s_{r,l,z} = \sum_z XF_{r,l,a} & \text{if} \quad \omega^m_{r,l} = \infty \end{cases}$$

(F-9) $$\begin{cases} NPF_{r,l,a} = \pi^w_{r,l,a} W^{ez}_{r,l,z} & \text{if} \quad \omega^m_{r,l} < \infty \quad \text{and} \quad a \in z \\ NPF_{r,l,a} = \pi^w_{r,l,a} W^e_{r,l} & \text{if} \quad \omega^m_{r,l} = \infty \end{cases}$$

Sector-specific factor markets

The sector specific factor—normally the natural resource base in natural resource sectors—is handled using an upward sloping supply curve with the elasticity given by ε^{ff} or by a logistic function with a specified maximum supply.[30] If the latter is infinite, the return to the sector

[29] Annex 5 describes how model equations are adjusted for inter-period gaps of greater than one year.
[30] A future version of the model will include a resource depletion module for natural gas and crude oil.

specific factor is assumed to rise at the same rate as the GDP deflator, see equation (F-11), else it is determined by market equilibrium. The finite supply curve has three shifters. The first, α^{fs}, is calibrated with base year data. The second, α^{rfs}, can be calibrated in a dynamic scenario to target a region specific variable, for example output or the regional producer price. The third, α^{gfs}, can be calibrated in a dynamic scenario to target a global variable, for example global output or the global price. In this case, the shifter moves each country/regional supply curve by the same proportional amount.

(F-10)
$$\begin{cases} XF_{r,a}^{s} = \alpha_{r,a}^{rfs}\alpha_{a}^{gfs}\alpha_{r,a}^{fs}\left(\dfrac{NPF_{r,NatRs,a}}{PGDPMP_{r}}\right)^{\varepsilon_{r,a}^{ff}} & \text{if} \quad \varepsilon_{r,a}^{ff} < \infty \quad \text{and} \quad XF_{r,a}^{max} = \infty \\ XF_{r,a}^{s} = \dfrac{XF_{r,a}^{max}}{1+\alpha_{r,NatRs,a}^{fs}e^{-\gamma_{r,a}^{fs}(NPF_{r,NatRs,a}/PGDPMP_{r})}} & \text{if} \quad \varepsilon_{r,a}^{ff} < \infty \quad \text{and} \quad XF_{r,a}^{max} = \infty \end{cases}$$

(F-11)
$$\begin{cases} XF_{r,a}^{s} = XF_{r,NatRs,a} & \text{if} \quad \varepsilon_{r,a}^{ff} < \infty \\ NPF_{r,NatRs,a,t} = PGDPMP_{r}.NPF_{r,NatRs,a,t-1} & \text{if} \quad \varepsilon_{r,a}^{ff} = \infty \end{cases}$$

Capital markets with the vintage capital specification

This section describes sectoral capital allocation under the assumption of multiple vintage capital. Capital market equilibrium under the vintage capital framework assumes the following:

- *New* capital is perfectly mobile and its allocation across sectors insures a uniform rate of return.
- *Old* capital in expanding sectors is equated to new capital, i.e. the rate of return on *Old* capital in expanding sectors is the same as the economy-wide rate of return on new capital.
- Declining sectors release *Old* capital. The released *Old* capital is added to the stock of *New* capital. The assumption here is that declining sectors will first release the most mobile types of capital, and this capital, being mobile, is comparable to *New* capital (e.g. transportation equipment).
- The rate of return on capital in declining sectors is determined by sector-specific supply and demand conditions.

The result of these assumptions is that if there are no sectors with declining economic activity, there is a single economy-wide rate of return. In the case of declining sectors, there will be an additional sector-specific rate of return on *Old* capital for each sector in decline.

To determine whether a sector is in decline or not, one assesses total sectoral demand (which of course, in equilibrium equals output). Given the capital-output ratio, it is possible to calculate whether the initially installed capital is able to produce the given demand. In a declining sector, the installed capital will exceed the capital necessary to produce existing demand. These sectors will therefore release capital on the secondary capital market in order to match their effective (capital) demand with supply. The supply schedule for released capital is a constant elasticity of supply function where the main argument is the change in the relative return between *Old* and *New* capital. Supply of capital to the declining sector is given by the following formula:

$$K_{a,Old}^{s} = K_a^0 \left[R_{a,Old} / R_{a,New} \right]^{\eta_i^k}$$

where K^s_{Old} is capital supply in the declining sector, K^0 is the initial installed (and depreciated) capital in the sector at the beginning of the period, and η^k is the dis-investment elasticity. (Note that in the model, the variable *R* is represented by *PF*.) In other words, as the rate of return on *Old* capital increases towards (decreases from) the rate of return on *New* capital, capital supply in the declining sector will increase (decrease). Released capital is the difference between K^0 and $K^{s,Old}$. It is added to the stock of *New* capital. In equilibrium, the *Old* supply of capital must equal the sectoral demand for capital:

$$K_{a,Old}^{s} = KV_{a,Old}$$

Inserting this into the equation above and defining the following variable

$$RR_a = R_{a,Old} / R_{a,New}$$

yields the following equilibrium condition:

$$KV_{a,Old} = K_a^0 \left[RR_a \right]^{\eta_a^k}$$

The supply curve is kinked, i.e. the relative rate of return is bounded above by 1. If demand for capital exceeds installed capital, the sector will demand *New* capital and the rate of return on *Old* capital is equal to the rate of return on *New* capital, i.e. the relative rate of return is 1. The kinked supply curve has been transformed into a mixed complementarity (MCP) relation. The following inequality is inserted in the model:

$$K_{a,Old}^{s} = K_a^0 \left[RR_a \right]^{\eta_a^k} \le K_a^{d,Not} = \chi_a^v XP_a$$

The right-hand side determines the *notional* demand for capital in sector *a*, i.e. it assesses aggregate output (equal to demand) and multiplies this by the capital output ratio for *Old* capital. This is then the derived demand for *Old* capital. If the installed capital is insufficient to meet demand for *Old* capital, the sector will demand *New* capital, and the inequality obtains with the relative rates of return capped at 1. If the derived demand for *Old* capital is less than installed capital, the sector will release capital according to the supply schedule. In this case the inequality transforms into an equality, and the relative rate of return is less than 1.

Equation (F-12) determines the capital output ratio, χ^v for *Old* capital. Equation (F-13) specifies the supply schedule of *Old* capital. In effect, this equation determines the variable *RR*, the relative rate of return between *Old* and *New* capital.

(F-12) $\chi_{r,a}^{v} = \dfrac{KV_{r,a,Old}}{XPv_{r,a,Old}}$

(F-13) $K_{r,a}^{0}\left(RR_{r,a}\right)^{\eta_i^k} \leq \chi_{r,a}^{v} XP_{r,a}$ and $RR_{r,a} \leq 1$

There is a single economy-wide rate of return on *New* capital. The equilibrium rate of return on *New* capital is determined by setting aggregate supply equal to aggregate demand. Aggregate demand for new capital is given by:

$$\sum_{a \in Expanding} \sum_{v} KV_{a,v}$$

where the set *Expanding* includes all sectors in expansion. Since *Old* capital in expanding sectors is equated with *New* capital, the appropriate sum is over all vintages. The aggregate capital stock of *New* capital is equal to the total capital stock, less capital supply in declining sectors:

$$K^{s} - \sum_{a \in Declining} K_{a}^{s,Old}$$

where the set *Declining* covers only those sectors in decline. However, at equilibrium, capital supply in declining sectors must equal capital demand for *Old* capital, and capital demand for *New* capital in these sectors is equal to zero. Hence, the supply of *Old* capital in declining sectors can be shifted to the demand side of the equilibrium condition for *New* capital, and this simplification yields equation (F-14) which determines the economy-wide rate of return on *New* capital. Equation (F-15) adds up capital demand across vintages. Equation (F-16) determines the vintage and sector specific rates of return.[31] For *New* capital, *RR* is 1 and thus the rate of return on *New* capital is always equal to the economy-wide rate of return (adjusted by the factor tax). For *Old* capital, if the sector is in decline, *RR* is less than 1 and the rate of return on *Old* capital will be less than the economy-wide rate of return (adjusted by the factor tax).

(F-14) $XF_{r,Captl,a} = \sum_{v} KV_{r,a,v}$

(F-15) $\sum_{a} XF_{r,Captl,a} = XFT_{r,Captl}$

(F-16) $PKV_{r,a,v} = PFT_{r,Captl} RR_{r,a} (1 + \tau_{r,Captl,a}^{v})$

Allocation of Output across Vintages

This section describes how output is allocated across vintages. Aggregate sectoral output, *XP*, is equated to aggregate sectoral demand and is derived from *XS*, which itself is derived from a CET aggregation of *XD* and *XET*. Given the beginning of period installed capital, it is possible to assess the level of *potential* output produced using the installed capital. If this level of output is

[31] These are the net rates of return after tax. Thus the relative rate of return variable, *RR*, is defined in terms of the net rate of return.

greater than the aggregate output (demand) level, the sector appears to be in decline, installed capital will be released, *Old* output will be equated with aggregate output (demand), and *New* output is zero. Equation (F-17) equates aggregate output, *XP*, to the sum of output across all vintages. Equation (F-18) determines output that can be derived from installed, or *Old* capital, thus equation (F-17) in essence determines output produced with *New* capital by residual. *Old* output is equated to the sectoral supply of *Old* capital, divided by the capital/output ratio. The final two equations are necessary price identities. Equation (F-19) sets the aggregate price of capital—in both declining and expanding sectors it is equal to the rate of return on *Old* capital. Equation (F-20) links the user-price of *all* factors of production to the after-tax sectoral price of each of the factors.

(F-17) $$XP_{r,a} = \sum_{a} XPv_{r,a,v}$$

(F-18) $$XPv_{r,a,Old} = K^0_{r,a}\left(RR_{r,a}\right)^{\eta_i^k} / \chi^v_{r,a}$$

(F-19) $$PF_{r,Captl,a} = PKV_{r,a,Old}$$

(F-20) $$PF_{r,fp,a} = (1+\tau^v_{r,fp,a})NPF_{r,fp,a}$$

Macro closure

Equation (M-1) defines the government accounting balance, S^g. It is the difference between revenues and expenditures, the latter including some share of stock-building expenditures. Equation (M-2) defines real government savings. Real government savings are fixed—to insure at the least debt sustainability. Nominal revenues are endogenous. The direct tax schedule shifts to achieve the given fiscal target (using the χ^k shifter). Equation (M-3) defines foreign savings, S^f. These are fixed in numéraire terms for all regions save a residual region (indexed by *rSav*). Equation (M-4) insures capital flow equilibrium at the global level (and in effect defines foreign savings for the residual region).[32] Equation (M-5) defines the depreciation allowance. Equation (M-6) represents the investment/savings balance, with aggregate gross investment expenditures on the left-side and total available savings on the right, including the depreciation allowance and adjusted for stock-building expenditures. The model's price anchor, or numéraire, *PNUM*, is defined in equation (M-7). It is defined as the unit value of manufactured exports from the high-income countries, where the set defined by *Numer* spans the manufactured sectors. One equation needs to be dropped from the model specification and typically one equation from equation (M-6) is dropped. This in fact represents a global Walras' Law that has global investment equal to global savings.

[32] Alternative versions of the model allow for partial mobility of global savings. These are described in Annex 4.

(M-1) $$S_r^g = YG_r - PC_{r,Gov} XC_{r,Gov} - \psi_{r,Gov}^{stb} \sum_i PA_{r,i,stb} XA_{r,i,stb}$$

(M-2) $$RSg_r = S_r^g / PGDPMP_r$$

(M-3) $$S_r^f = \bar{S}_r^f .PNUM \quad \text{for} \quad r \notin rSav$$

(M-4) $$\sum_r S_r^f \equiv 0$$

(M-5) $$DeprY_r = \delta_r PC_{r,Inv} K_{r,t}$$

(M-6) $$PC_{r,Inv} XC_{r,Inv} = \sum_h S_{r,h}^h + S_r^g + S_r^f + DeprY_r - \psi_{r,Inv}^{stb} \sum_i PA_{r,i,stb} XA_{r,i,stb}$$

(M-7) $$PNUM = \sum_{r \in HIC} \sum_{r'} \sum_{i \in Numer} \phi_{r,r',i}^n WPE_{r,r',i}$$

The following block of equations provides the main macroeconomic identities. Equations (M-8) and (M-9) represent nominal and real GDP at market price, *GDPMP* and *RGDPMP* respectively. The GDP at market price deflator, *PGDPMP*, is defined in equation (M-10). Per capita real output, *RGDPPC*, is defined in equation (M-11). Equation (M-12) defines real per capita income growth. And the GDP absorption shares, *GDPShr*, are provided in equation (M-13). Equation (M-14) defines real domestic absorption—it is the sum of household, government and investment real expenditures.

The default closure rules of the model are as follows:

- Household savings are endogenous and are either driven by the demographic-influenced savings function or as part of the ELES consumer demand system.[33]

- Government revenues are endogenous and government expenditures, as a share of nominal GDP, are fixed, thus total expenditures are endogenous. The government balance is fixed, in part to avoid problems of financing sustainability. The government balance is achieved with a uniform shift in the household direct tax schedule. This implies that new revenues, for example generated by a carbon tax, would lower direct taxes paid by households.

- Investment is savings driven. Household and government savings were discussed above. Foreign savings, in the default closure are fixed. Thus investment is largely influenced through household savings.[34]

[33] The latter may allow for demographics and other factors to influence the ELES parameters between periods in the dynamic setting where ELES parameters may be re-calibrated.

[34] Alternative closures are conceivable, for example targeting investment (as a share of GDP) and allowing the household savings schedule adjust to achieve the target.

- The current account, the mirror entry of the capital account, is exogenous. Ex ante changes to trade, for example a rise in the world price of imported oil, is met through ex post changes in the real exchange rate.

(M-8)
$$\begin{aligned} GDPMP_r &= \sum_i \left[\sum_{in} PA_{r,i,in} XA_{r,i,in} + PA_{r,i,stb} XA_{r,i,stb} \right] + \sum_{ih} NT_{r,ih} \\ &+ \sum_{r'} \sum_i \left[WPE_{r,r',im} WTF_{r,r',im} - WPM_{r',r,im} WTF_{r',r,im} \right] + PTMG_r . XTMG_r \end{aligned}$$

(M-9)
$$\begin{aligned} RGDPMP_r &= \sum_i \left[\sum_{in} PA_{r,i,in,0} XA_{r,i,in} + PA_{r,i,stb,0} XA_{r,i,stb} \right] \\ &+ \sum_{r'} \sum_i \left[WPE_{r,r',im,0} WTF_{r,r',im} - WPM_{r',r,im,0} WTF_{r',r,im} \right] \\ &+ \sum_{ih} PW_{ih,0} (XET_{r,ih} - XMT_{r,ih,0}) + PTMG_{r,0} . XTMG_r \end{aligned}$$

(M-10) $PGDPMP_r = GDPMP_r / RGDPMP_r$

(M-11) $RGDPPC_r = RGDPMP_r / Pop_r$

(M-12) $g_{r,t}^{ypc} = \left(\dfrac{RGDPPC_{r,t}}{RGDPPC_{r,t-n}} \right)^{1/n} - 1$

(M-13) $GDPShr_{r,in} = \dfrac{PC_{r,in} XC_{r,in}}{GDPMP_r}$

(M-14) $RYD_{r,t} = \sum_{in} PC_{r,in,0} XC_{r,in,t}$

Model Dynamics

Model dynamics are driven by three factors—similar to most neo-classical growth models. Population and labor force growth rates are exogenous and given essentially by the UN Population Division scenario. The labor force growth rate is equated to the growth rate of the working age population, i.e. the population aged between 15 and 64.[35]

The second factor is capital accumulation. The aggregate capital stock in any given year, *KStock*, is equated to the previous year capital stock, less depreciation at a rate of δ, plus the previous period's volume of investment, XC_{Inv}, see equation (G-1). The latter is influenced by the national savings rate plus foreign savings and, as well, the unit cost of investment. The aggregate capital stock variable takes two forms. The first, *KStock*, is the aggregate capital stock evaluated at \$2004 prices. The second is the 'normalized' aggregate capital stock, *XFT*. The normalized

[35] In future work, these assumptions will be linked to other variables influencing both the decision to work (i.e. the labor force participation rate) and the skill level (via assumptions on education).

capital stock is equal to the tax inclusive base year capital remuneration, i.e. the user cost of capital across sectors. It is normalized because its price is set to 1 in the base year. The ratio of the normalized capital stock to the actual capital stock provides a measure of the gross rate of return to capital. It is assumed that both measures of the capital stock grow at the same rate and hence equation (G-2) that equalizes the ratio of the two measures.[36]

(G-1)	$KStock_r = (1-\delta_r).KStock_{r,-1} + XC_{r,Inv,-1}$
(G-2)	$XFT_{r,Captl} = (XFT_{r,Captl,0} / KSTock_{r,0})KStock_r$
(G-3)	$\lambda^f_{r,l,a} = (1+\pi_{r,a}+\gamma^l_r).\lambda^f_{r,l,a,-1}$

The third factor is productivity. There are a number of productivity factors peppered throughout the model. The key productivity factor is λ^f that corresponds to factor productivity. The following assumptions are made regarding productivity:

- Sectors are segmented into three groups—agriculture, manufacturing and services.
- Productivity in agriculture is exogenous and factor neutral. The λ^n and λ^v parameters are set to grow at some exogenous and uniform rate.
- In the other sectors, productivity is labor augmenting only—and is uniform across both skilled and unskilled labor.
- There is a wedge between productivity in manufacturing and services, represented by the factor π in equation (G-3). It is typically assumed that productivity in manufacturing is greater than in services, i.e. π for manufacturing is positive, and it is zero for services.
- In the calibration, or business-as-usual scenario, the uniform productivity factor, γ^l, is calibrated to achieve some target level of per capita growth, at least for some period, including historical validation from the base year to some current year (say from 2004 to 2009), and including some medium term horizon such as 2015. After 2015, the parameter γ^l can be fixed and per capita growth then is an endogenous variable. In most policy scenarios, the γ^l parameter is fixed.
- Energy efficiency is assumed to improve at some exogenous rate that influences the λ^e parameter.
- International trade and transport margins, τ^{tm}, are assumed to improve at some exogenous rate.

Emissions , climate and impact modules

The module's sequence is as follows. First total emissions are derived. The current version of the model includes four greenhouse gases—carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O) and the fluoridated gases as an aggregate (F-gases). Though most of the emissions are linked to intermediate and final demand, i.e. the consumption of some emitting good or service, in production some may also be linked to capital (e.g. cattle stock in the case of methane), land (in the case of methane and nitrous oxide emissions in agriculture) and/or aggregate output (e.g.

[36] It is important to use the actual capital stock in the capital accumulation function since the level of investment must correspond to the actual capital stock, not the normalized level.

municipal waste-base methane emissions). The emissions of greenhouse gases lead to atmospheric concentrations—emissions directly add to the atmosphere, but concentrations in the atmosphere also interact with the ocean and land, creating a dynamic process that would continue even in the absence of emissions. The atmospheric concentration has an impact on radiative forcing, i.e. how much of the sun's energy is reflected back to space. Finally, there is a set of equations that links radiative forcing to temperature global mean temperature change. The final phase of the module links changes in the average mean temperature to economic impacts that feed back into production and demand thereby closing the loop between economic activities, climate, back to economic activities.

Greenhouse gas emissions

The first emissions equation, equation (C-1), determines the level of emissions, *EMI*, of type *em* for each unit of consumption of commodity *i* by agent *aa*, where *aa* covers all production activities and final demand accounts. It is simply a fixed coefficient with respect to the demand level. The emissions rate, ρ, can be adjusted in the baseline by the factor χ to allow for autonomous improvements in the emission rates.[37] Equation(C-2) captures emissions linked to the use of factors of production such as capital and/or land. Equation (C-3) are emissions linked to generic production activities and not to a specific technology, i.e. they are simply output based emissions. The aggregate emission by region (or country *r*), *EMITot*, is defined in equation (C-4) and is the double sum over all agents and sources (consumption, factor use and production level), with the possibility of an additional exogenous level of emissions, *EMIOth*. The level of global emissions, *EMIGbl*, is the summation across all countries and regions, with an additional exogenous component not accounted for in the regional models—see equation (C-5).

(C-1) $$EMI_{r,em,i,aa} = \chi^e_{em}\rho_{r,em,i,aa}XA_{r,i,aa}$$

(C-2) $$EMI_{r,em,fp,a} = \chi^e_{em}\rho_{r,em,fp,a}XF_{r,fp,a}$$

(C-3) $$EMI_{r,em,Total,a} = \chi^e_{em}\rho_{r,em,Total,a}XP_{r,a}$$

(C-4) $$EMITot_{r,em} = \sum_{aa}\sum_{is} EMI_{r,em,is,aa} + EMIOth_{r,em}$$

(C-5) $$EMIGbl_{em} = \sum_{r} EMITot_{r,em} + EMIOthGbl_{em}$$

Emission taxes, caps and trade

There are a number of different potential regimes to limit carbon emissions. The simplest is simply to impose a carbon tax, i.e. set the variable τ^{emi} to some value (measured as \$2004 per unit of emitted C). Emission caps can be set on either a single region/country basis, with a differentiated carbon tax across regions/countries, or on a region-wide basis with a uniform

[37] This has only been used to calibrate the emissions rate of non-CO_2 greenhouse gases.

carbon tax. Quota regions are indexed by *rq* and can be assigned one or more countries. Examples of cap and cap and trade scenarios are provided in Annex 7. Equation (C-6) implements emissions caps for each coalition of regions subject to a cap (potentially just a single country). The sum of emissions across all regions belong to region *rq* is capped to *EMICap* (the shifter is explained below). Equation (C-6) determines the regional emissions tax, τ^{emiR}, which will be uniform across all countries/regions belonging to the coalition region. Equation (C-7) then is an accounting identity that equates the country/region tax, τ^{emi}, to the region-wide emissions tax.

(C-6) $$\sum_{r \in rq} EMITot_{r,em} = \chi_{em}^{Cap} EMICap_{rq,em}$$

(C-7) $$\tau_{r,em}^{emi} = \tau_{rq,em}^{emiR}$$

(C-8) $$QuotaY_{r,em}^{emi} = \tau_{r,em}^{emi} \left[EMIQuota_{r,em} - EMITot_{r,em} \right] \quad \text{if Cap and Trade is active}$$

The shifter in equation (C-6) allows for additional targeting, for example a cap on global emissions. Say for example one wants to cap global emissions by 20 percent but only impose a cap on Annex I emissions. There is some potential leakage from the cap on Annex I countries—with non-Annex I countries increasing their emissions—because the world price of fossil fuels may decline and because they increase their production of carbon intensive goods for export to the now less competitive Annex I markets. The cap on Annex I countries can then be thought of as setting the burden shares across Annex I countries and the shifter, χ^{Cap}, in equation (C-6) is then endogenous to meet the overall objective, for example capping global emissions.

Equation (C-8) determines the value of the trade in emissions quota when country/region specific quotas, *EMIQuota*, are allocated. The value of the quota is the difference between the quota and actual emissions, *EMITot*, valued at the emissions tax level. Currently, it is assumed that the quota rents are recycled back to the government.

Concentration, forcing and temperature

The current version of ENVISAGE uses a highly simplified climate module that is largely inspired by the climate module in the MERGE model.[38] It replaces the original climate module that was based on Nordhaus' DICE 2007 model[39] because the latter has a CO_2-only focus and ENVISAGE needed a module that could handle other greenhouse gases. We may eventually also assess the implementation of the PAGE09 climate module that has the added advantage of providing spatially distinct temperature change in part linked to regional differences in latitude.[40]

Greenhouse gases are treated differently in their impacts on temperature. Carbon emissions are released into the atmosphere that is divided into five boxes, *Box*, indexed by *b*. New emissions

[38] As described in Manne et al 1995 and implemented in the GAMS version of MERGE [need to check source of code and updates].
[39] Nordhaus 2008, also described in greater detail in Annex 6.
[40] Hope 2010.

are released into the five boxes (equation C-9), where the fraction parameter, φ, sums to 1.[41] The level of carbon in each of the boxes decays over time at the rate δ^d (that is box-specific). Equation (C-10) then determines the total concentration of atmospheric carbon (or the stock) in gtC, *Conc*, summing over all of the boxes and added to the pre-industrial concentration. For the other greenhouse gases, indexed by *xghg*, the atmospheric concentration is equal to the previous period's concentration with a decay parameter δ^x, to which is added new emissions, equation (C-11). The total concentration is the sum of the transient concentration, $Conc^t$, to which is added some equilibrium stock, equation (C-12).

(C-9) $$Box_{b,t} = \delta^d_{CO2,b} Box_{b,t-1} + \varphi^b_b EMIGbl_{CO2,t}$$

(C-10) $$Conc_{CO2,t} = \sum_b Box_{b,t} + PIC_{CO2}$$

(C-11) $$Conc^t_{xghg,t} = \delta^x_{xghg,t} Conc^t_{xghg,t-1} + EMIGbl_{xghg,t}$$

(C-12) $$Conc_{xghg,t} = Conc^t_{xghg,t} + EqConc_{xghg,t}$$

(C-13) $$RF_{CO2,t} = \rho^f_{CO2} \ln\left(Conc_{CO2,t} / Conc_{CO2,0}\right)$$

(C-14) $$RF_{xghg,t} = \zeta^f_{xghg} \rho^f_{xghg} \left[\left(\frac{\chi_{xghg} Conc_{xghg,t}}{GWP_{xghg,t}} \right)^{\omega^f_{xghg}} - \left(\frac{\chi_{xghg} Conc_{xghg,0}}{GWP_{xghg,t}} \right)^{\omega^f_{xghg}} \right]$$

(C-15) $$Temp^{eq}_t = \rho^t \left[RF_0 + \sum_{ghg} RF_{ghg,t} \right]$$

(C-16) $$Temp_t = (1-\lambda^t) Temp_{t-1} + \lambda^t Temp^{eq}_{t-1} - \rho^t \left(Cool_t - Cool_0 + RF^X_t \right)$$

The radiative forcing impact, *RF*, of carbon concentration is a logarithmic function of the concentration level where ρ^f is a critical parameter that determines the climate sensitivity—typically measured as the radiative forcing impact of a doubling of CO_2 concentration relative to the pre-industrial level, equation (C-13). The radiative forcing impacts of the other greenhouse gases is normally captured by a power equation in the difference in concentration (from base levels) where the power is either the square root, or linear, equation (C-14). The concentrations measured in Envisage are in gtCeq and are converted back to millions of tons using the global warming potential conversion factor, *GWP*. The ζ parameter captures atmospheric chemical interaction effects across the different greenhouse gases.

The actualized mean global surface temperature lags behind the potential temperature change as it takes time for atmosphere and ocean temperature transfer. Equation (C-15) captures the potential temperature impact, $Temp^{eq}$, of the changes in radiative forcing which is a linear

41 More details on the underlying theory, parameterization, and handling of the multi-step time periods is provided in Annex 6.

function of the aggregate change in radiative forcing. The actual change in temperature, *Temp,* is a weighted average of the previous temperature change and the potential temperature—with potential adjustments due to exogenous cooling and radiative forcing (e.g. sulfates), equation (C-16).

Climate change economic impacts

The incorporation of climate-related impacts in models of climate change has largely been relegated to highly aggregate economic models (Nordhaus 1994, 2001 and 2008, Hope 2006) using a macro damage function that link changes in temperature to a percentage impact on productivity—normally with an assumption of non-linearity. Hope's damages were initially split into three distinct impacts—macroeconomic, non economic (such as eco-systems), and a third damage linked to a sudden discontinuity that could happen after a given temperature threshold. Hope 2010 has added a fourth channel that splits the impact of sea level rise from the macroeconomic damage function. Nordhaus 2010 has similarly split his macroeconomic damage function into two components with sea level rise split from the rest of the impacts.[42] The FUND model (Anthoff and Tol 2008) is also a macro model, but they have vastly extended the impact side to include agriculture, forestry, water resources, energy consumption, sea level rise, eco-systems, human health and extreme weather. The initial version of ENVISAGE only incorporated agricultural damages—calibrated to estimates in Cline 2007, but this limited impact has been superseded by a new and more complete set of impact estimates and described below.[43]

Impacts are based on a 2-dimensional table of impact *sources* and impact *destinations*. The impact sources take into account the following:

sea	Sea level rise
agr	Agricultural productivity
wat	Water availability
onj	On the job (labor) productivity
tou	Tourism
hhe	Human health[44]
end	Energy demand

The following impact destinations are considered:

42 The critical part of sea level rise is that the lagged structure of temperature exchange between the atmosphere and the sea is unusually long so that even if atmospheric temperature rise is reduced relatively rapidly, the impact on sea level rise would take centuries to dissipate.

43 Much of the remainder of this section is based on Roson 2009.

44 At the moment, the human health component only reflects the direct effect on labor productivity, and not the increased demand for health services.

lp	Labor productivity (or stock)
kp	Capital productivity (or stock)
tp	Land productivity (or stock)
mp	Multi-factor productivity
hc	Household consumption of energy
hcser	Household consumption of market services
incab	Income from abroad

The bulk of the impacts are assumed to be linear with respect to temperature change and are summarized by equation (C-17), where *ddam* is the specific damage function by source and destination. It is implemented as a deviation from the no-damage situation, where *ddam* takes the value of 1 in the absence of climate change. A quadratic damage function is used in the case of agriculture—affecting multi-factor productivity, as depicted in equation (C-18). The following set of equations implements the damages directly on the relevant model variables. Equation (C-19) implements the (quadratic) damage on the top-level productivity parameter in the crop sectors, i.e. it is a uniform shift in the production possibilities frontier across all inputs. It enters in equations (P-1) through (P-3).[45] The next set of four equations determines the impacts on the factors of production in efficiency units. Equations (C-20) through (C-22) determine the cumulative impact on respectively labor, land and capital. The standard productivity factors, λ^f, are determined in the dynamics module, and the climate-impact adjusted parameters, λ^{gf}, enter the production functions. Equation (C-23) is a simple identity as, for the moment, it is assumed that climate change does not have an impact on the availability of natural resources (i.e. fossil fuels).

Equation (C-24) represents the impact on household demand. The impact is assumed to affect the minimal subsistence bundle as represented by the θ parameter. The impact parameter is calibrated to the impact on overall consumption, hence the impact on the subsistence level is scaled for the share of the subsistence level in the overall level of consumption (per capita).

Finally, equation (C-25) represents the impact on tourism revenues, *iit*. This is a linear function of the temperature change, where iit_0 represents base year tourism revenues. Tourism revenues accrue to households and this requires a change to equation (Y-10).[46]

Parametrization of the damage functions is described in Annex 8.

45 The parameters of the damage function are region specific and reflect to some extent the base year structure of agricultural production, however, the damage as currently formulated applies uniformly across all crop sectors.

46 Similar to all variables that deal with households, the income is allocated across households using the χ^{iit} allocation vector—but for the moment, ENVISAGE has only a single representative household and thus the parameter has unit value.

(C-17)	$ddam_{r,src,dst,t} = 1 + \chi^{ccd}_{r,src,dst,t}\left(T_t - T_0\right) \quad \text{for} \quad src \in Linear$
(C-18)	$ddam_{r,agr,mp,t} = 1 + \alpha^{a1}_r \min(1, T_t - T_0) + \alpha^{a2}_r\left(T_t - T_0\right) + \alpha^{a3}_r\left(T_t - T_0\right)^2$
(C-19)	$\delta^{cd}_{r,cr} = ddam_{r,agr,mp}$
(C-20)	$\lambda^{gf}_{r,l,a} = \lambda^{f}_{r,l,a}\prod_{src} ddam_{r,src,lp}$
(C-21)	$\lambda^{gf}_{r,LandR,a} = \lambda^{f}_{r,LandR,a}\prod_{src} ddam_{r,src,tp}$
(C-22)	$\lambda^{gf}_{r,Captl,a} = \lambda^{f}_{r,Captl,a}\prod_{src} ddam_{r,src,kp}$
(C-23)	$\lambda^{gf}_{r,natrs,a} = \lambda^{f}_{r,natrs,a}$
(C-24)	$\theta^{gh}_{r,k,h} = \theta^{h}_{r,k,h}\left[1 + \frac{\left(ddam_{r,end,hc} sign(\theta^{h}_{r,k,h}) - 1\right)}{\theta^{h}_{r,k,h} / \left(HX_{r,k,h,0} / Pop_{r,h,0}\right)}\right]$
(C-25)	$iit_{r,h} = \chi^{iit}_{r,h} iit_{r,0}\left(T_t - T_0\right)$

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Annex 1: The CES/CET function

The CES Function

Because of the frequent use of the constant elasticity of substitution (CES) function, this annex develops some of the properties of the CES, including some of its special cases. The CES function can be formulated as a cost minimization problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$

subject to

$$V = \left[\sum_i a_i \left(\lambda_i X_i \right)^{\rho} \right]^{1/\rho}$$

where V is the aggregate volume (of production, for example), X are the individual components ("inputs") of the production function, P are the corresponding prices, and a and λ are technological parameters. The a parameters are most often called the share parameters. The λ parameters are technology shifters. The parameter ρ is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

A bit of algebra produces the following derived demand for the inputs, assuming V and the prices are fixed:

$$(1) \qquad X_i = \alpha_i \left(\lambda_i \right)^{\sigma-1} \left(\frac{P}{P_i} \right)^{\sigma} V$$

where we define the following relationships:

$$\rho = \frac{\sigma - 1}{\sigma} \Leftrightarrow \sigma = \frac{1}{1-\rho} \qquad \text{and} \qquad \sigma \geq 0$$

$$\alpha_i = a_i^{\sigma} \Leftrightarrow a_i = \alpha_i^{1/\sigma}$$

and

$$(2) \qquad P = \left[\sum_i \alpha_i \left(\frac{P_i}{\lambda_i} \right)^{1-\sigma} \right]^{1/(1-\sigma)} \Leftrightarrow P.V = \sum_i P_i X_i$$

P is called the CES dual price, it is the aggregate price of the CES components. The parameter σ, is called the substitution elasticity. This term comes from the following relationship, which is easy to derive from Equation (1):

$$\frac{\partial \left(X_i / X_j \right)}{\partial \left(P_i / P_j \right)} \frac{\left(P_i / P_j \right)}{\left(X_i / X_j \right)} = -\sigma$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note, we are assuming that the substitution elasticity is a positive number).

For example, if the price of input *i* increases by 10 per cent with respect to input *j*, the ratio of input *i* to input *j* will decrease by (around) σ times 10 per cent.

We can also derive some key elasticities from these relations. First, is the elasticity of the aggregate price with respect to one of the input prices:

$$(3) \qquad \frac{\partial P}{\partial P_i} . \frac{P_i}{P} = s_i = \frac{P_i . X_i}{P . X}$$

In other words, the percent change in the aggregate price is equal to the percent change in the component price multiplied by the value share of that component represented by s_i.

The price elasticities, holding volume constants are given by the following formula:

$$(4) \qquad \varepsilon_{ij} = \frac{\partial X_i}{\partial P_j} \frac{P_j}{X_i} = \sigma s_j - \sigma \delta_{ij} = \sigma (s_j - \delta_{ij})$$

This implies that all components are gross substitutes.

The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero, in other words, there is no substitution between inputs, no matter what the input prices are. Equations (1) and (2) become:

$$(1') \qquad X_i = \frac{\alpha_i V}{\lambda_i}$$

$$(2') \qquad P = \sum_i \alpha_i \frac{P_i}{\lambda_i}$$

The aggregate price is the weighted sum of the input (efficient) prices. The Cobb-Douglas function is for the special case when σ is equal to one. It should be clear from Equation (2) that this case needs special handling. The following equations provide the relevant equations for the Cobb-Douglas:

$$(1'') \qquad X_i = \alpha_i \frac{P}{P_i} V$$

$$(2'') \qquad P = A^{-1} \prod_i \left(\frac{P_i}{\alpha_i \lambda_i} \right)^{\alpha_i}$$

where the production function is given by:

$$V = A \prod_i (\lambda_i X_i)^{\alpha_i}$$

and

$$\sum_i \alpha_i = 1$$

Note that in Equation (1") the value share is constant, and does not depend directly on technology change.

Calibration

Typically, the base data set along with a given substitution elasticity are used to calibrate the CES share parameters. Equation (1) can be inverted to yield:

$$\alpha_i = \left(\frac{P_i}{P}\right)^{\sigma} \frac{X_i}{V}$$

assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalized to 1, simplifying the above expression to a true value share. Let's take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalized to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by α, not the share parameters represented by *a*. This saves on compute time since the *a* parameters never appear explicitly in any equation, whereas a raised to the power of the substitution elasticity, i.e. α, occurs frequently.)

The CET Function

With less detail, the following describes the relevant formulas for the CET function, which is similar to the CES specification.

$$\max \sum_i P_i X_i$$

subject to

$$V = \left[\sum_i g_i X_i^{\nu}\right]^{1/\nu}$$

where *V* is the aggregate volume (e.g. aggregate supply), *X* are the relevant components (sector-specific supply), *P* are the corresponding prices, *g* are the CET (primal) share parameters, and ν is the CET exponent. The CET exponent is related to the CET transformation elasticity, ω via the following relation:

$$\nu = \frac{\omega + 1}{\omega} \Leftrightarrow \omega = \frac{1}{\nu - 1}$$

Solution of this maximization problem leads to the following first order conditions:

$$X_i = \gamma_i \left(\frac{P_i}{P}\right)^{\omega} V$$

$$P = \left[\sum_i \gamma_i P_i^{1+\omega}\right]^{1/(1+\omega)}$$

where the γ parameters are related to the primal share parameters, *g*, by the following formula:

$$\gamma_i = g_i^{-\omega} \Leftrightarrow g_i = \left(\frac{1}{\gamma_i}\right)^{1/\omega}$$

Annex 2: The demand systems

The model contains four different possible demand systems for determining household demand for goods and services:

- CDE or constant differences in elasticities—largely derived from the GTAP model
- ELES or extended linear expenditure system
- LES or linear expenditure system
- AIDADS of an implicitly directly additive demand system, an extension of the LES that allows for more plausible Engel behavior

The default demand system is the AIDADS system that in theory has better long-term properties than the other three as defined by the shape of the Engel curves. The core model documentation describes implementation of the LES/ELES and AIDADS as all three have similar core equations. The implementation of the CDE will be described at the end of the CDE section.

Three of the demand systems (CDE, LES, AIDADS) use a two-tiered structure to first allocate income between savings and expenditures on goods and services. The ELES integrates the savings allocation within its specification. All four systems determine the demand for consumer goods that are different from produced goods. A transition matrix approach is subsequently used to convert consumer goods into produced goods.[47]

The CDE demand system

The *Constant Difference of Elasticities* (CDE) function is a generalization of the CES function, but it allows for more flexibility in terms of substitution effects across goods.[48] The starting point is an implicitly additive indirect utility function (see Hanoch 1975) from which we can derive demand using Roy's identity (and the implicit function theorem).

General Form

A dual approach is used to determine the properties of the CDE function. The indirect utility function is defined implicitly via the following expression:

$$(1) \qquad V(p,u,Y) = \sum_{i=1}^{n} \alpha_i u^{e_i b_i} \left(\frac{p_i}{Y} \right)^{b_i} \equiv 1$$

where p is the vector of commodity prices, u is utility, and Y is income. Using Roy's identity and the implicit function theorem[49] we can derive demand, x, where v is the indirect utility function (defined implicitly):

47 At the moment, for lack of additional data, the transition matrix is largely diagonal, with the exception of energy goods.

48 More detailed descriptions of the CDE can be found in Hertel et al (1991), Surry (1993) and Hertel (1997).

49 See Varian 1992, p. 109.

(2) $$x_i = -\frac{\partial v}{\partial p_i} / \frac{\partial v}{\partial Y} = -\left(\frac{\partial V}{\partial p_i} / \frac{\partial V}{\partial u}\right) / \left(\frac{\partial V}{\partial Y} / \frac{\partial V}{\partial u}\right) = -\left(\frac{\partial V}{\partial p_i} / \frac{\partial V}{\partial Y}\right)$$

This then leads to the following demand function—that is implemented as equation (25) in the model implementation.

(3) $$x_i = \frac{\alpha_i b_i u^{e_i b_i} \left(\frac{p_i}{Y}\right)^{b_i - 1}}{\sum_j \alpha_j b_j u^{e_j b_j} \left(\frac{p_j}{Y}\right)^{b_j}}$$

Elasticities

In order to calibrate the CDE system, it is necessary to derive the demand and income elasticities of the CDE. The algebra is tedious, but straightforward.

The own-price elasticity is given by the following:

(4) $$\varepsilon_i = \frac{\partial x_i}{\partial p_i} \frac{p_i}{x_i} = -\frac{e_i b_i s_i}{\sum_j s_j e_j} + (b_i - 1) + \frac{s_i \sum_j s_j e_j b_j}{\sum_j s_j e_j} - s_i b_i$$

In deriving the elasticity, we make use of the following formula that defines the elasticity of utility with respect to price (and again makes use of the implicit function theorem):

(5) $$\frac{\partial u}{\partial p_i} \frac{p_i}{u} = -\frac{p_i}{u} \left(\frac{\partial V}{\partial p_i}\right) / \left(\frac{\partial V}{\partial u}\right) = -\frac{s_i}{\sum_j s_j e_j}$$

The price elasticity of utility is approximately the value share of the respective demand component as long as the weighted sum of the expansion parameters, *e*, is close to unity. The value share is defined in the next equation:

(6) $$s_i = \frac{p_i x_i}{Y}$$

Letting $\sigma_i = 1 - b_i$ (or $b_i = 1 - \sigma_i$), we can also write:

(7) $$\varepsilon_i = s_i \left[\sigma_i - \frac{e_i (1 - \sigma_i)}{\sum_j s_j e_j} - \frac{\sum_j s_j e_j \sigma_j}{\sum_j s_j e_j} \right] - \sigma_i$$

With σ uniform, we also have:

(8) $$\varepsilon_i = -\frac{s_i e_i (1 - \sigma)}{\sum_j s_j e_j} - \sigma$$

With both *e* and σ uniform, the formula simplifies to:

(9) $$\varepsilon_i = -s_i(1-\sigma) - \sigma = \sigma(s_i - 1) - s_i$$

Equation (9) reflects the own-price elasticity for the standard CES utility function. Finally, with *e* uniform but not σ, we have:

(10) $$\varepsilon_i = s_i\left[2\sigma_i - 1 - \sum_j s_j\sigma_j\right] - \sigma_i$$

The derivation of the cross elasticities is almost identical and will not be carried out here. Combining both the own-and cross price elasticities, the matrix of substitution elasticities takes the following form where we use the Kronecker product, δ:[50]

(11) $$\varepsilon_{ij} = s_j\left[-b_j - \frac{e_i b_i}{\sum_k s_k e_k} + \frac{\sum_k s_k e_k b_k}{\sum_k s_k e_k}\right] + \delta_{ij}(b_i - 1)$$

Again, we replace *b* by 1-σ, to get:

(12) $$\varepsilon_{ij} = s_j\left[\sigma_j - \frac{e_i(1-\sigma_i)}{\sum_k s_k e_k} - \frac{\sum_k s_k e_k \sigma_k}{\sum_k s_k e_k}\right] - \delta_{ij}\sigma_i$$

For uniform σ, equation (22) takes the form:

(13) $$\varepsilon_{ij} = -\frac{e_i s_j(1-\sigma)}{\sum_k s_k e_k} - \delta_{ij}\sigma$$

And with a uniform *s* and *e*, we have:

(14) $$\varepsilon_{ij} = -s_j(1-\sigma) - \delta_{ij}\sigma = \sigma(s_j - \delta_{ij}) - s_j$$

Finally, for a uniform *e* only, the matrix of elasticities is:

(15) $$\varepsilon_{ij} = s_j\left[\sigma_j - (1-\sigma_i) - \sum_k s_k\sigma_k\right] - \delta_{ij}\sigma_i$$

The income elasticities are derived in a similar fashion:

(16) $$\eta_i = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i} = \frac{1}{\sum_k s_k e_k}\left[e_i b_i - \sum_k s_k e_k b_k\right] - (b_i - 1) + \sum_k b_k s_k$$

For this, we need the elasticity of utility with respect to income:

(17) $$\frac{\partial u}{\partial Y}\frac{Y}{u} = -\frac{Y}{u}\left(\frac{\partial V}{\partial Y}\right)\Big/\left(\frac{\partial V}{\partial u}\right) = \frac{1}{\sum_k s_k e_k}$$

Note that for a uniform and unitary *e*, the income elasticity of utility is 1.

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50 δ takes the value of 1 along the diagonal (i.e. when $i=j$) and the value 0 off-diagonal (i.e. when $i \neq j$).

Replacing *b* with 1-σ, equation (16) can be re-written to be:

(18) $$\eta_i = \frac{1}{\sum_k s_k e_k}\left[e_i(1-\sigma_i) + \sum_k s_k e_k \sigma_k\right] + \sigma_i - \sum_k s_k \sigma_k$$

With a uniform σ, the income elasticity becomes:

(19) $$\eta_i = \frac{1}{\sum_k s_k e_k}\left[e_i(1-\sigma) + \sigma \sum_k s_k e_k\right] = \frac{e_i(1-\sigma)}{\sum_k s_k e_k} + \sigma$$

With *e* uniform, the income elasticity is unitary, irrespective of the values of the σ parameters.

From the Slutsky equation, we can calculate the compensated demand elasticities:

(20) $$\xi_{ij} = \varepsilon_{ij} + s_j \eta_i = -\delta_{ij}\sigma_i + s_j\left[\sigma_j + \sigma_i - \sum_k s_k \sigma_k\right]$$

The cross-Allen partial elasticity is equal to the compensated demand elasticity divided by the share:

(21) $$\sigma_{ij}^a = \sigma_j + \sigma_i - \sum_k s_k \sigma_k - \delta_{ij}\sigma_i / s_j$$

It can be readily seen that the difference of the partial elasticities is constant, hence the name of *constant difference in elasticities*.

(22) $$\sigma_{ij}^a - \sigma_{il}^a = \sigma_j - \sigma_l$$

With a uniform σ, we revert back to the standard CES where there is equivalence between the CES substitution elasticity and the cross-Allen partial elasticity:

(23) $$\sigma_{ij}^a = \sigma$$

Calibration

Calibration assumes that we know the value shares, the own uncompensated demand elasticities and the income elasticities. The weighted sum of the income elasticities must equal 1, so the first step in the calibration procedure is to make sure Engel's law holds. One alternative is to fix some (or none) of the income elasticities and re-scale the others using least squares. The problem is to minimize the following objective function:

$$\sum_{i \in \Omega}\left(\eta_i - \eta_i^0\right)^2$$

subject to

$$\sum_{i\in\Omega} s_i\eta_i = 1 - \sum_{i\notin\Omega} s_i\eta_i$$

where the set Ω contains all sectors where the income elasticity is not fixed, i.e. its complement contains those sectors with fixed income elasticities. The solution is:

$$\eta_i = \eta_i^0 + s_i \frac{1 - \sum_{i\notin\Omega} s_i\eta_i - \sum_{i\in\Omega} s_i\eta_i^0}{\sum_{i\in\Omega} s_i^2} \quad \forall i \in \Omega$$

Calibration of the σ parameters is straightforward given the own elasticities and the input value shares. The first step is to calculate the Allen partial elasticities, these are simply the own elasticities divided by the budget shares:

(24) $$\sigma_{ii}^a = \frac{\varepsilon_{ii}}{s_i}$$

Next, equation (21) is setup in matrix form:

(25) $$\sigma_{ii}^a = A\sigma_i$$

where the matrix A has the form:

(26) $$A = \begin{bmatrix} 2 - \frac{1}{s_1} - s_1 & -s_2 & \dots & -s_n \\ -s_1 & 2 - \frac{1}{s_2} - s_2 & \dots & -s_n \\ \vdots & \vdots & \ddots & \vdots \\ -s_1 & -s_2 & \dots & 2 - \frac{1}{s_n} - s_n \end{bmatrix}$$

or each element of *A* has the following formula:

$$a_{ij} = \delta_{ij}(2 - 1/s_i) - s_j$$

We can then solve for σ:

(27) $$\sigma_i = A^{-1}\sigma_{ii}^a$$

There is nothing which guarantees the consistency of the calibrated σ parameters, which are meant to be positive. The calculation of the σ parameters depends only on the budget shares and the own-price uncompensated elasticities. If the calibrated σ parameters are not all positive, one could modify the elasticities until consistency is achieved. In practice, problems have occurred when a sector's budget share dominates total expenditure.

The *e* parameters are derived from Equation (19) and normalizing them so that their share weighted sum is equal to 1. Equation (19) can then be converted to matrix form and inverted:

(28) $$B = \begin{bmatrix} s_1\sigma_1 + (1-\sigma_1) & s_2\sigma_2 & \dots & s_n\sigma_n \\ s_1\sigma_1 & s_2\sigma_2 + (1-\sigma_2) & \dots & s_n\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ s_1\sigma_1 & s_2\sigma_2 & \dots & s_n\sigma_n + (1-\sigma_n) \end{bmatrix}$$

or

$$b_{ij} = s_j\sigma_j + \delta_{ij}(1-\sigma_i)$$

(29) $$e_i = B^{-1}C_i = B^{-1}\left(\eta_i - \sigma_i + \sum_k s_k\sigma_k\right)$$

Calibration of the α parameters is based on equations (1) and (3). Start first with equation (3) and write it in terms relative to consumption of good 1, i.e.:

(30) $$\frac{x_i}{x_1} = \frac{\alpha_i b_i u^{e_i b_i}\left(\dfrac{p_i}{Y}\right)^{b_i-1}}{\alpha_1 b_1 u^{e_1 b_1}\left(\dfrac{p_1}{Y}\right)^{b_1-1}}$$

This equation can be used to isolate α_i:

(31) $$\alpha_i = \frac{x_i}{x_1}\frac{\alpha_1 b_1 u^{e_1 b_1}\left(\dfrac{p_1}{Y}\right)^{b_1-1}}{b_i u^{e_i b_i}\left(\dfrac{p_i}{Y}\right)^{b_i-1}}$$

and then inserted into equation (1):

(32) $$\sum_{i=1}^{n}\alpha_i u^{e_i b_i}\left(\frac{p_i}{Y}\right)^{b_i} = \alpha_1 u^{e_1 b_1}\frac{b_1}{s_1}\left(\frac{p_1}{Y}\right)^{b_1}\left[\sum_{i=1}^{n}\frac{s_i}{b_i}\right] \equiv 1$$

The final expression in equation (32) can be used to solve for α_1 since the formula must equal 1 by definition:

(33) $$\alpha_1 = u^{-e_1 b_1}\frac{s_1}{b_1}\left(\frac{Y}{p_1}\right)^{b_1}\left[\sum_{i=1}^{n}\frac{s_i}{b_i}\right]^{-1}$$

Substituting back into equation (31) we get:

(34) $$\alpha_i = \frac{x_i}{b_i} u^{-e_i b_i} \left(\frac{Y}{p_i} \right)^{b_i - 1} \left[\sum_{j=1}^{n} \frac{s_j}{b_j} \right]^{-1}$$

The final calibration expression is then the following:

(35) $$\alpha_i = \frac{s_i}{b_i} \left(\frac{Y}{p_i} \right)^{b_i} \frac{u^{-e_i b_i}}{\sum_{j=1}^{n} \frac{s_j}{b_j}}$$

Utility is undefined in the base data and it is easiest to simply set it to 1.

In conclusion, for calibration we need the budget shares, initial prices, total expenditure, income elasticities and the own-price uncompensated elasticities. From this, we can derive base year consumption volumes, the Allen partial substitution elasticities through equation (24), σ (and therefore *b*) through equation (27) and the inversion of the A-matrix , *e* through equation (29) and inversion of the B-matrix, and finally α through equation (35).

The following block of equations replace equations (D-5) through (D-8) when using the CDE implementation of consumer demand (*ifCDE*=1). Equations (D-5) and (D-6) reflect the first order conditions of the CDE. Equation (D-5') is an auxiliary equation that simplifies equation (D-6') that is the actual demand equation and determines the demand variable *HX*. Demand is specified on a per capita basis so total private expenditure is divided by population to define per capita expenditure and per capita demand is then re-multiplied by population to get total private demand. Equation (D-8') represents the (implicit) utility function for the CDE, where *PHX* are consumer prices.

(D-5') $$\Theta_{r,h} = \sum_k \alpha^h_{r,k,h} b^h_{r,k,h} U_{r,h}^{e^h_{r,k,h} b^h_{r,k,h}} \left(\frac{PHX_{r,k,h}}{(YC_{r,h} / Pop_{r,h})} \right)^{b^h_{r,k,h}}$$

(D-6') $$HX_{r,k,h} = Pop_{r,h} \frac{\alpha^h_{r,k,h} b^h_{r,k,h} U_{r,h}^{e^h_{r,k} b^h_{r,k}} \left(\frac{PHX_{r,k,h}}{(YC_{r,h} / Pop_{r,h})} \right)^{b^h_{r,k,h} - 1}}{\Theta_{r,h}}$$

(D-8') $$\sum_k \alpha^h_{r,k,h} U_{r,h}^{e^h_{r,k,h} b^h_{r,k,h}} \left(\frac{PHX_{r,k,h}}{(YC_{r,h} / Pop_{r,h})} \right)^{b^h_{r,k,h}} \equiv 1$$

The ELES demand system

Many models assume separability in household decision making between saving and current consumption. Lluch and Howe[51] introduced a relatively straightforward extension of the LES to include the saving decision simultaneously with the allocation of income on goods and services, this has become known as the extended linear expenditure system or the ELES. The ELES is based on consumers maximizing their intertemporal utility between a bundle of current consumption and an expected future consumption bundle represented in the form of savings. The ELES has several attractive features. The utility function of the ELES has the following form:

$$(36) \qquad u = \prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s}$$

with

$$(37) \qquad \sum_i \mu_i + \mu_s = 1$$

where u is utility, x is the vector of consumption goods, S is household saving (in value), P^s is the price of saving, and μ and θ are ELES parameters.

The consumer solves the following problem:

$$(38) \qquad \max \prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s}$$

subject to

$$(39) \qquad \sum_{i=1}^{n} p_i x_i + S = Y$$

where p is the vector of consumer prices, and Y is disposable income. The demand functions are:

$$(40) \qquad x_i = \theta_i + \frac{\mu_i}{p_i}\left(Y - \sum_{j=1}^{n} p_j \theta_j\right)$$

$$(41) \qquad S = \mu_s \left(Y - \sum_{j=1}^{n} p_j \theta_j\right) = Y - \sum_{j=1}^{n} p_j x_j$$

The term in parentheses is sometimes called supernumerary income, i.e. it is the income that remains after subtracting total expenditures on the so-called subsistence (or floor) expenditures as represented by the θ parameter. The parameter μ then represents the marginal budget share out of supernumerary income.

From the demand equation we can derive the income and price elasticities:

51 See Lluch (1973) and Howe (1975).

(42) $$\eta_i = \frac{\mu_i Y}{p_i x_i} = \frac{\mu_i}{s_i} \qquad \eta_s = \frac{\mu_s Y}{S} = \frac{\mu_s}{s}$$

(43) $$\varepsilon_i = \frac{\theta_i(1-\mu_i)}{x_i} - 1 \qquad \varepsilon_s = -1$$

(44) $$\varepsilon_{ij} = -\frac{\mu_i \theta_j p_j}{p_i x_i} = -\frac{\mu_i \theta_j p_j}{s_i Y} \qquad \varepsilon_{sj} = -\frac{\mu_s \theta_j P_j}{sY} = -\frac{\theta_j P_j}{Y^*}$$

where s is the average propensity to save. Note that the matrix of elasticities can be collapsed to a single formula using the Kronecker factor:

(45) $$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y}\left[\delta_{ij} Y^* + p_j \theta_j\right]$$

where δ takes the value 1 when i equals j and Y^* is supernumerary income.

Welfare

With the addition of saving, the indirect utility function is given by:

(46) $$v(p,Y) = \prod_i \left(\frac{\mu_i}{p_i} Y^*\right)^{\mu_i} \left(\frac{\mu_s}{P^s} Y^*\right)^{\mu_s}$$

or

(47) $$v(p,Y) = \frac{Y^*}{P} \qquad \text{where} \quad P = \prod_i \left(\frac{p_i}{\mu_i}\right)^{\mu_i} \left(\frac{P^s}{\mu_s}\right)^{\mu_s}$$

The expenditure function is derived by minimizing the cost of achieving a given level of utility, u. It is set-up as:

$$\min \sum_{i=1}^{n} p_i x_i + S$$

subject to

$$\prod_i (x_i - \theta_i)^{\mu_i} \left(\frac{S}{P^s}\right)^{\mu_s} = u$$

The final expression for the expenditure function is:

(48) $$E(p,u) = \sum_{i=1}^{n} p_i \theta_i + uP$$

where

(49) $$P = \prod_i \left(\frac{p_i}{\mu_i}\right)^{\mu_i} \left(\frac{P^s}{\mu_s}\right)^{\mu_s}$$

Calibration

Calibration of the ELES uses the budget share information from the base SAM, including the household saving share. Typically, calibration uses income elasticities for all of the *n* commodities represented in the demand system and uses equation (42) to derive the marginal budget shares, μ_i. This procedure leads to a residual income elasticity, which in this case is the income elasticity of saving. The derived savings income elasticity may be implausible, in which case adjustments need to be made to individual income elasticities for the goods, or adjustments can be made on the group of goods, assuming some target for the savings income elasticity.

The first step is therefore to calculate the marginal budget shares using the average budget shares and the initial income elasticity estimates.

$$\mu_i = \frac{\eta_i p_i x_i}{Y} = \eta_i s_i$$

The savings marginal budget share is derived from the consistency requirement that the marginal budget shares sum to 1:

$$\mu_s = 1 - \sum_{i=1}^{n} \mu_i$$

Assuming this procedure leads to a plausible estimate for the savings income elasticity, the next step is to calibrate the subsistence minima, θ. This can be done by seeing that the demand equations, (40), are linear in the θ parameters. Note that in the case of the ELES the system of equation are of full rank because the μ parameters do not sum to 1 (over the *n* commodities. They only sum to 1 including the marginal saving share. This may lead to calibration problems if the propensity to save is 0, which may be the case in some SAMs with poor households.) The linear system can be written as:

$$C = I\theta + MY - M\ \Pi\ \theta$$

where I is an n x n identity matrix, M is a diagonal matrix with μ_i / P_i on the diagonal, and Π is a matrix, where each row is identical, each row being the transpose of the price vector. The above system of linear equations can be solved via matrix inversion for the parameter θ:

$$\theta = A^{-1}C^*$$

where

$$A = I - M\ \Pi$$

$$C^* = C - MY$$

The matrices A and C^* are defined by:

$$A = [a_{ij}] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i \dfrac{p_j}{p_i} & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \frac{\mu_i Y}{p_i}$$

The A and C^* matrices are simplified if the price vector is initialized at 1:

$$A = [a_{ij}] = \begin{cases} 1 - \mu_i & \text{if} \quad i = j \\ -\mu_i & \text{if} \quad i \neq j \end{cases}$$

$$C^* = [c_i] = x_i - \mu_i Y$$

In GAMS one could invert the system of equations embodied in equation (40) directly by solving for the endogenous θ while holding all of the other variables and parameters fixed.

The AIDADS demand system

Both the CDE and the ELES suffer from relatively poor Engel behavior. In the case of the CDE, income elasticities stay relatively constant at their initial level irrespective of income growth. The ELES has even worse properties as it relatively quickly converges towards a Cobb-Douglas utility function with unitary income elasticities for all goods, even allowing for a population-adjustment to the subsistence parameters. An alternative demand system, known as AIDADS, has received more attention recently[52] and was initially proposed by Rimmer and Powell.[53] It is a relatively natural extension to the LES function, the latter being a special case of the AIDADS function. The insight of Rimmer and Powell was to allow the marginal propensity term of the LES to be a function of other variables, rather than be a constant as in the LES. This allows for more complex demand behavior, as well as providing better validation for observed changes in consumption patterns.

Basic formulation

AIDADS starts with the implicitly additive utility function given by:

(50) $$\sum_i U_i(x_i, u) \equiv 1$$

Assume the following functional form for the utility function:

(51) $$U_i = \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right)$$

where

[52] Hertel and co-authors have been using AIDADS with the GTAP model. See for example Yu et al. (2002).

[53] See Rimmer and Powell (1992a), Rimmer and Powell (1992b) and Rimmer and Powell (1996).

(52) $$\mu_i = \frac{\alpha_i + \beta_i G(u)}{1 + G(u)}$$

with the restrictions

$$\sum_i \alpha_i = \sum_i \beta_i = 1$$

$$0 \le \alpha_i \le 1$$

$$0 \le \beta_i \le 1$$

$$\theta_i < x_i$$

Cost minimization implies the following:

$$\min \sum_i p_i x_i$$

subject to

(53) $$\sum_i \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right) \equiv 1$$

The first order conditions lead to:

(54) $$\lambda \frac{\partial U_i}{\partial x_i} = p_i = \lambda \frac{\mu_i}{x_i - \theta_i} \Rightarrow \lambda\mu_i = p_i x_i - p_i \theta_i$$

summing over *i* and using the fact that the μ_i sum to unity implies:

(55) $$\lambda = \sum_i p_i x_i - \sum_i p_i \theta_i = Y - \sum_i p_i \theta_i = Y^*$$

where Y is aggregate expenditure, and Y^*, sometimes referred to as supernumerary income, is residual expenditure after subtracting total expenditure on the so-called subsistence minima, θ.

Re-inserting equation (55) into (54) yields the consumer demand equations:

(56) $$x_i = \theta_i + \frac{\mu_i}{p_i} Y^* = \theta_i + \frac{\mu_i}{p_i}\left[Y - \sum_j p_j \theta_j\right]$$

Equation (56) is virtually identical to the ELES demand equation (40) above. Similar to the linear expenditure system (LES), demand is the sum of two components—a subsistence minimum, θ, and a share of supernumerary income. Unlike the LES, the share parameter, μ, is not constant, but depends on the level of utility itself. AIDADS collapses to the LES if each β parameter is equal to the corresponding α parameter, with the ensuing function of utility, $G(u)$, dropping from equation (52).

Elasticities

This section develops the main expressions for the income and price elasticities. These formulas will be needed to calibrate the initial parameters of the AIDADS function.

Income elasticity
To derive further properties of AIDADS requires specifying a functional form for $G(u)$. Rimmer and Powell (1996) propose the following:

$$G(u) = e^u \tag{57}$$

The first step is to calculate the marginal budget share, ρ, defined as:

$$\rho_i = p_i \frac{\partial x_i}{\partial Y}$$

The following expression can be derived from equation (56):

$$\frac{\partial x_i}{\partial Y} = \frac{Y^*}{p_i}\frac{\partial \mu_i}{\partial Y} + \frac{\mu_i}{p_i}\frac{\partial Y^*}{\partial Y} = \frac{Y^*}{p_i}\frac{\partial \mu_i}{\partial u}\frac{\partial u}{\partial Y} + \frac{\mu_i}{p_i}$$

Thus:

$$\rho_i = \mu_i + Y^* \frac{\partial \mu_i}{\partial u}\frac{\partial u}{\partial Y} \tag{58}$$

Expression (58) can be expanded in two steps—first evaluating the partial derivative of the share variable, μ, with respect to utility, and then the more difficult calculation of the partial derivative of u with respect to income. The share formula is:

$$\mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}$$

Its derivative is:

$$\frac{\partial \mu_i}{\partial u} = \frac{\left(1+e^u\right)\left(\beta_i e^u\right) - \left(\alpha_i + \beta_i e^u\right)e^u}{(1+e^u)^2} = \frac{e^u\left(\beta_i - \alpha_i\right)}{(1+e^u)^2} \tag{59}$$

Utility and income are combined in implicit form and thus we will invoke the implicit function theorem to calculate the partial derivative of u with respect to Y. First, insert equation (56) into equation (53):

$$\sum_i \mu_i \ln\left(\frac{x_i - \theta_i}{Ae^u}\right) = \sum_i \mu_i \ln\left(\frac{\mu_i Y^*}{Ae^u p_i}\right) = 1$$

Expanding the latter expression yields:

$$f\left(u, Y\right) = \sum_i \mu_i \ln\left(\frac{\mu_i}{p_i}\right) + \ln\left(Y^*\right) - \ln\left(A\right) - u = 1 \tag{60}$$

which provides the implicit relation between Y and u. The implicit function theorem states the following:

(61) $$\frac{\partial u}{\partial Y} = -\frac{\partial f}{\partial Y}\left[\frac{\partial f}{\partial u}\right]^{-1}$$

The partial derivative of *f* with respect to *Y* is simply:

(62) $$\frac{\partial f}{\partial Y} = \frac{1}{Y^*}$$

The partial derivative of *f* with respect to *u* is:

(63) $$\begin{aligned}
\frac{\partial f}{\partial u} &= -1 + \sum_i \left[\frac{\partial \mu_i}{\partial u}\ln\left(\frac{\mu_i}{p_i}\right) + \mu_i \frac{p_i}{\mu_i} p_i \frac{\partial \mu_i}{\partial u}\right] \\
&= -1 + \frac{e^u}{(1+e^u)^2}\sum_i \left[\left(\ln\left(\frac{\mu_i}{p_i}\right)+1\right)(\beta_i - \alpha_i)\right] \\
&= \frac{e^u}{(1+e^u)^2}\left[\sum_i (\beta_i - \alpha_i)\ln(x_i - \theta_i) - \frac{(1+e^u)^2}{e^u}\right] \\
&= \frac{e^u}{(1+e^u)^2}\Omega^{-1}
\end{aligned}$$

where

(64) $$\Omega = \left[\sum_i (\beta_i - \alpha_i)\ln(x_i - \theta_i) - \frac{(1+e^u)^2}{e^u}\right]^{-1}$$

The second line uses equation (59). In the third line, equation (56) substitutes for the expression in the logarithm, and the adding up constraint allows for the deletion of non-indexed variables. Substituting (62) and (63) into (61) yields:

(65) $$\frac{\partial u}{\partial Y} = -\frac{\Omega}{Y^*}\frac{(1+e^u)^2}{e^u}$$

Substituting (59) and (65) into (58) yields:

$$\rho_i = \mu_i - (\beta_i - \alpha_i)\Omega$$

The income elasticities are derived from the following expression:

(66) $$\eta_i = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i} = \frac{\partial x_i}{\partial Y}\frac{Y}{x_i}\frac{p_i}{p_i} = \frac{\rho_i}{s_i}$$

where s_i is the average budget share:

(67) $$s_i = \frac{p_i x_i}{Y} = \frac{p_i \theta_i}{Y} + \mu_i \frac{Y^*}{Y}$$

It can also be written as:

(68) $$s_i = \mu_i + \left(\frac{p_i\theta_i - \mu_i \sum_j p_j\theta_j}{Y} \right)$$

Thus the income elasticity, η, is equal to the ratio of the marginal budget share, ρ, and the average budget share, *s*.

Price elasticity

The matrix of substitution elasticities is identical to the expression for the ELES and has the form:

(69) $$\sigma_{ij} = \left[\mu_j - \delta_{ij}\right] \frac{\mu_i Y^*}{s_i s_j Y}$$

where

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

It is clear that the matrix is symmetric. The matrix of substitution elasticities is also equal to:

(70) $$\sigma_{ij} = \left[\mu_j - \delta_{ij}\right] \frac{\mu_i Y^*}{s_i s_j Y} = \frac{(x_i - \theta_i)}{x_i} \frac{(x_j - \theta_j)}{x_j} \frac{Y}{Y^*} - \frac{\delta_{ij}}{s_j} \frac{(x_i - \theta_i)}{x_i}$$

The compensated demand elasticities derive from the following:

(72) $$\xi_{ij} = s_j \sigma_{ij} = \left[\mu_j - \delta_{ij}\right] \frac{\mu_i Y^*}{s_i Y}$$

Finally, the matrix of uncompensated demand elasticities is given by:

(73) $$\varepsilon_{ij} = \xi_{ij} - s_j \eta_i = \left[\mu_j - \delta_{ij}\right] \frac{\mu_i Y^*}{s_i Y} - s_j \eta_i$$

The uncompensated demand elasticities can also be written as:

(73') $$\varepsilon_{ij} = -\frac{\mu_i}{s_i Y}\left[p_j\theta_j + \delta_{ij} Y^*\right] + \frac{s_j}{s_i}(\beta_i - \alpha_i)\Omega$$

The first term on the right-hand side is always negative. The second term differs from the LES expression for the uncompensated demand elasticities.[54] We can see from expression (73') that the AIDADS specification allows for both gross complementarity and substitution. As well, it allows for luxury goods, i.e. positive own-price demand elasticities should the second term be positive and greater than the first term.

54 Recall that for the LES, the α and β terms are equal and thus the second term drops.

Implementation

Implementation of AIDADS is somewhat more complicated than the LES since the marginal propensity to consume out of supernumerary income is endogenous, and utility is defined implicitly. The following four equations are needed for model implementation:

$$(74) \qquad Y^* = Y - \sum_i p_i \theta_i$$

$$(75) \qquad x_i = \theta_i + \frac{\mu_i}{p_i} Y^*$$

$$(76) \qquad \mu_i = \frac{\alpha_i + \beta_i e^u}{1 + e^u}$$

$$(77) \qquad u = \sum_i \mu_i \ln(x_i - \theta_i) - 1 - \ln(A)$$

Equations (74) and (75) are identical to their LES (ELES) counterparts. Equation (76) determines the level of the marginal propensity to consume out of supernumerary income, μ, which is a constant in the case of the LES (ELES). It requires however the calculation of the utility level, *u*, which is defined in equation (77).

Calibration

[To be updated] Calibration requires more information than the LES. Where the LES has 2*n* parameters to calibrate (subject to consistency constraints), AIDADS has 3*n* parameters (less the consistency requirements)—α, β and θ. The calibration system includes equations (74)-(77) which have 2+2*n* endogenous variables (Y^*, θ, μ, and *A*). There are no equations for calibrating the α and β parameters. If we have knowledge of the income elasticities, we can add the following equations:

$$(78) \qquad \Psi = \frac{1}{\Omega} = \left[\sum_i (\beta_i - \alpha_i) \ln(x_i - \theta_i) - \frac{(1 + e^u)^2}{e^u} \right]$$

$$(79) \qquad \eta_i = \frac{\rho_i}{s_i} = \frac{\mu_i - (\beta_i - \alpha_i)\Omega}{s_i} = \frac{\mu_i}{s_i} - \frac{(\beta_i - \alpha_i)}{s_i \Psi}$$

There are an additional 1+*n* equations, solving for Ψ and α. There is need for an additional *n* equations. Assuming we have knowledge of at least *n* price elasticities, for example the own-price elasticities, we can add the following equation:

$$(80) \qquad \varepsilon_{ii} = -\frac{\mu_i}{s_i Y} \left[p_i \theta_i + Y^* \right] + (\beta_i - \alpha_i)\Omega$$

The α and β parameters are not independent, the following restrictions must hold:

(81) $$\sum_i \alpha_i = 1$$

(82) $$\sum_i \beta_i = 1$$

The system is under-determined, there are 5+4*n* equations and 3+4*n* variables. One solution, is to make the own-price elasticities endogenous. In this case, we are adding *n* variables, but then the system is over-determined. We can minimize a loss function with respect to the price elasticities:

(83) $$L = \sum_i \left(\varepsilon_i - \varepsilon_i^0\right)^2$$

where ε^0 represents an initial guess of the own-price elasticities and the calibration algorithm will calculate the endogenous ε in order to minimize the loss function, subject to constraints (78)-(82) and the model equations (74)-(77). The exogenous parameters in the calibration procedure include p, x, s, Y, η, ε^0 and u.

Annex 3: Alternative trade specification

The GTAP database decomposes aggregate demand for goods and services by agent *a* into a domestic component and an (aggregate) import component. It is this possible to implement that Armington specification at the agent level—though the default model uses a national Armington specification, in part to (significantly) reduce the size of the model. This short annex describes how the model needs to be modified to allow for agent-specific behavior. Equations (T-1) through (T-6) in the model would be replaced by the equations in the block below. Equations (T-1') and (T-1'') define the Armington domestic and import components at the agent level, where all share parameters and substitution elasticities are agent-specific. Equation (T-2') defines the agent-specific Armington price. And equations (T-4') and (T-5') determine the aggregate domestic demand for domestic production and imports respectively. Under this specification, the variables *XAT* and *PAT* are dropped. (N.B. This specification has not been reviewed for use with the 'energy aware' version of ENVISAGE. Among other potential issues, there are no γ parameters that allow for the adding up of energy volumes in efficiency units.)

(T-1') $$XD_{r,i,a} = \alpha^d_{r,i,a}\left(\frac{PA_{r,i,a}}{(1+\tau^{Ad}_{r,i,a})PD_{r,i}}\right)^{\sigma^m_{r,i,a}} XA_{r,i,a}$$

(T-1'') $$XM_{r,i,a} = \alpha^m_{r,i,a}\left(\frac{PA_{r,i,a}}{(1+\tau^{Am}_{r,i,a})PMT_{r,i}}\right)^{\sigma^m_{r,i,a}} XA_{r,i,a}$$

(T-2') $$PA_{r,i,a} = \left[\alpha^d_{r,i,a}\left((1+\tau^{Ad}_{r,i,a})PD_{r,i}\right)^{1-\sigma^m_{r,i,a}} + \alpha^m_{r,i,a}\left((1+\tau^{Am}_{r,i,a})PMT_{r,i}\right)^{1-\sigma^m_{r,i,a}}\right]^{1/(1-\sigma^m_{r,i,a})}$$

(T-4') $$XDT^d_{r,i} = \sum_a XD_{r,i,a}$$

(T-5') $$XMT_{r,i} = \sum_a XM_{r,i,a}$$

Annex 4: Alternative capital account closures

[To be completed]

…has three different closures for the capital account. The simplest is simply to fix the capital account at base year levels. The second option, as described in HT, is to allow the capital account equilibrate changes in the expected rate of return to capital across regions, i.e. the percentage change of regional rates of return are equal. If returns are equal initially, this is equivalent to assuming perfect international capital mobility. The third option, also described in HT, assumes that the 'global' investor has an optimal portfolio initially, and adjusts capital flows to maintain the same portfolio *ex post*.

Equation (58) defines the average rate of return to capital in each region, *AvgRoR*. It is the weighted average of the sectoral rates of return. [? Should the weights be fixed, i.e. indexed by t0 ?]. The current net rate of return, *RoRC*, is then defined as the average gross regional rate of return, adjusted by changes to the unit cost of capital, and less depreciation—equation (59). Equation (60) defines the motion equation for aggregate capital. The end-of-period capital stock, K_{t+1}, is equal to the beginning period capital stock, K_t, adjusted for depreciation, and augmented by the current period's volume of investment, XC_{Inv}. The expected rate of return, *RoRE*, is assumed to decline with positive additions to the capital stock. This is the motivation behind equation (61). [See HT for a more detailed description.] Equation (62) defines the value of net investment, *NInv*. Equation (63) defines the average global rate of return, *RoRG*.

The three foreign capital closure rules are encapsulated in equation (64) and are driven by a model flag labeled *KFlowFlag*. The first rule is simply to fix the capital account. To preserve model homogeneity, the initial volume is multiplied by the model numéraire to provide a nominal foreign saving. The second rule equates the percentage change in the expected rate of return in each region. The third rule assumes that global investment is allocated across regions such that the regional composition of capital stocks is invariant. This implies that the percent change in net investment is equal across regions [Shouldn't we be using as a rule that the capital stock in value terms is proportionately the same across regions]. Equation (64) is defined for all regions except for one. The left out region is indexed by *RSAV* that is a subset of the set of regions, *r*. Closure of the model is guaranteed by equation (65) that forces the global sum of the capital flows to be identically equal to zero.

Annex 5—Dynamic model equations with multi-step time periods

Dynamics in a multi-year step

The step size in the model scenarios are allowed to vary across time—in order to save compute time and storage. Particularly in the long-run scenarios, annual increments are not particularly useful. Some of the equations in the model—essentially almost any equation that relies on a lagged variable need to take into account the variable step size, for example equation (G-1), the capital accumulation equation.

$$KStock_r = (1-\delta_r).KStock_{r,-1} + XC_{r,Inv,-1}$$

In fact, this equation is not even necessary in the model for a step size of 1 since both variables on the right-hand side of the equation are lags. However, let *n* be the step-size, eventually 1. Then through recursion, the capital accumulation function becomes:

$$KStock_t = (1-\delta)^n KStock_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1} XC_{Inv,t-j}$$

If the model is run in step sizes greater than 1, the intermediate values of real investment are not calculated. They can be replaced by assuming a linear growth model for investment:

$$XC_{Inv,t} = (1+\gamma^I)XC_{Inv,t-1}$$

Replacing this in the accumulation function yields:

$$KStock_t = (1-\delta)^n KStock_{t-n} + \sum_{j=1}^{n}(1-\delta)^{j-1}(1+\gamma^I)^{n-j} XC_{Inv,t-n}$$

With some algebraic manipulation (that is done for a number of similar expressions below), this formula can be reduced to the following:

$$KStock_t = (1-\delta)^n K_{t-n} + \frac{(1+\gamma^I)^n - (1-\delta)^n}{\gamma^I + \delta} XC_{Inv,t-n}$$

Where we have the following equation to determine the growth rate of investment:

$$XC_{Inv,t} = (1+\gamma^I)^n XC_{Inv,t-n}$$

which itself is now a function of contemporaneous investment. If *n* is equal to 1, it is clear that this equation simplifies to the simple 1 step accumulation function. The capital accumulation function is no longer exogenous since it depends on the investment growth rate, which itself is endogenous. To avoid scale problems, equations (G-1a) and (G-1b) are used in place of (G-1) to provide the *n*-step capital stock accumulation function. Equation (G-1a) is likely to evaluate to somewhere between 10 and 20 since the first term is 1 plus the average annual growth of investment, to which is added the depreciation rate less 1. If investment growth is 5% and depreciation is also 5%, then the value is 10. The first term on the right-hand side of equation (G-1b) is likely to be relatively small since it takes the previous capital stock and subtracts a multiple of the previous period's investment (lagged *n* years), and then multiplies by the depreciation factor, so that the largest term is the second term, which is a multiple of the current volume of investment.

(G-1a)	$IGFact_{r,t} = \left[\left(\frac{XC_{r,Inv,t}}{XC_{r,Inv,t-n}} \right)^{1/n} - 1 + \delta \right]^{-1}$
(G-1b)	$KStock_{r,t} = \left[KStock_{r,t-n} - IGFact_{r,t} XC_{r,Inv,t-n} \right] (1 - \delta_{r,t})^n + IGFact_{r,t} XC_{r,Inv,t}$

The savings function, equation (D-8) also needs modification in a dynamic scenario with multiple years between solution periods. The new equation (D-8) below shows the modification of the savings function, where the new variables are g^{PLT15} and g^{P65UP} that represent the average annual growth rates of the youth and elderly dependency ratios.

(D-8)

$$\begin{aligned} s_{r,t}^{s} &= \chi_r^s \alpha_r^s \frac{1 - (\beta_r^s)^{n+1}}{1 - \beta_r^s} + (\beta_r^s)^{n+1} s_{r,t-n}^{s} + \beta_r^g g_{r,t}^{pc} \frac{1 - (\beta_r^s / (1 + g_r^{pc}))^{n+1}}{1 - \beta_r^s / (1 + g_r^{pc})} \\ &+ \beta_r^y DRAT_{r,t}^{PLT15} \frac{1 - (\beta_r^s / (1 + g_{r,t}^{PLT15}))^{n+1}}{1 - \beta_r^s / (1 + g_{r,t}^{PLT15})} \\ &+ \beta_r^e DRAT_r^{P65UP} \frac{1 - (\beta_r^s / (1 + g_{r,t}^{P65UP}))^{n+1}}{1 - \beta_r^s / (1 + g_{r,t}^{P65UP})} \end{aligned}$$

Annex 6: Climate modules

DICE 2007 Climate module

This Annex describes the climate module used in the DICE 2007 model.[55] It was the climate module used in the original version of ENVISAGE. It was replaced by the MERGE climate module in order to handle the non-CO2 greenhouse gases. While not in current use, its description may be of use to interested readers and the annex also shows how the DICE module has been extended to cover any time definition—not the fixed 10-year time steps of the DICE model.

The model contains three sinks for CO_2 emissions—the atmosphere and the upper and deep oceans. These three sinks are indexed by *z*. In each period, there is a flow of carbon across the three sinks using a 3 x 3 transition matrix, K. Each column of the transition matrix represents the share of the stock in the sink that flows to a different sink. Thus the diagonal element represents the share of the stock that stays in its own sink. The current values of the concentration transition matrix are provided in a more detail below.

(1) $$Conc_z = \mathrm{K}.Conc_{z,-1} + EMIGbl_{z,CO2,-1}$$

(2) $$Forc_{atmos} = fCO2x.\frac{\log_{10}(Conc_{atmos} / ConcPI)}{\log_{10}(2)} + ForcOth$$

(3) $$Temp_{zt} = \mathrm{T}.Temp_{zt,-1} + \Theta.Forc_{zt}$$

Equation (1) determines the concentration level in each sink. The concentration level is equal to its lagged value, multiplied by the transition matrix. In the absence of new emissions, one can determine the long-term equilibrium by multiplying the matrix K *n*-times, where *n* is large enough that the transition matrix converges towards a constant matrix. Carbon emissions are entirely added to atmospheric concentration.[56] Note that emissions in the model are in terms of carbon. To convert to CO2, multiply the carbon emissions by the factor (44/12).

Equation (2) converts atmospheric concentrations to its impact on radiative forcing. Forcing is a logarithmic function (based 10) of concentration with two key parameters. The first is the pre-industrial concentration level, *ConcPI*. The second is the amount of forcing induced by a doubling of concentration from its pre-industrial level, $fCO2x$. The relation allows for an exogenous amount of forcing, that could eventually be negative, as is the current case, due to SO_2 emissions.

Temperature, measured as the increment to temperature in °C since 1900, like concentration, has interactions between the atmosphere and the oceans. In this case the ocean is treated as a single

55 See Nordhaus (2008).

56 The variable *EMIGbl* is a vector defined overall all sinks, but emissions to the two ocean sinks are always 0.

sink and the subset *zt* of *z* covers only *atmos* and *dpocn*. Equation (3) provides the link between temperature in the two sinks with their previous respective temperatures, through a transition matrix T, and the incremental impact from forcing through the matrix Θ.[57] The temperature transition and forcing matrices are further developed below.

The transition matrices in the DICE model are based on a fixed 10-year time step between years. In the ENVISAGE model, the time gap is variable. The model therefore requires two modifications to the DICE version of the climate module. First, it is necessary to convert the 10-year transition matrices to a single-year transition matrix, and then to code the dynamic equations to allow for variable gap dynamic expression.

Emissions and concentration

In the DICE model, the 10-year concentration transition matrix B has the following form and values:

$$\mathrm{B} = \left\{\begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & b_{11} & b_{12} & b_{13} \\ upocn & b_{21} & b_{22} & b_{23} \\ dpocn & b_{31} & b_{32} & b_{33} \end{array}\right\} = \left\{\begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & 0.810712 & 0.097213 & 0 \\ upocn & 0.189288 & 0.852787 & 0.003119 \\ dpocn & 0 & 0.050000 & 0.996881 \end{array}\right\}$$

Nearly 19% of atmospheric carbon is absorbed by the upper sea (over a decade), and the upper sea releases about 10% of its carbon to the atmosphere (over a decade).

If emissions end at some point *T*, then the equilibrium concentration of carbon can be given by the following equation:

$$Conc_{\infty} = \mathrm{B}^{\infty} Conc_{T}$$

The equilibrium B matrix, B^{∞}, is given by:

$$\mathrm{B}^{\infty} = \left\{\begin{array}{cccc} & atmos & upocn & dpocn \\ atmos & 0.029269 & 0.029269 & 0.029269 \\ upocn & 0.056991 & 0.056991 & 0.056991 \\ dpocn & 0.913739 & 0.913739 & 0.913739 \end{array}\right\}$$

This implies that in the long run the atmosphere will contain just under 3% of total carbon in all three physical zones (or sinks) as of the terminal year of emissions, with about 6% in the upper ocean and the remaining 91% absorbed in the deep ocean. At today's level of carbon concentrations we would get the following equilibrium concentration levels (assuming all emissions stop today):[58]

---

[57] The variable *Forc* is a vector defined over both sinks but is only non-zero for the atmosphere.

[58] The concentration is expressed at the stock of carbon (C) in gigatons.

$$Conc_{\infty} = \begin{bmatrix} 598 \\ 1{,}164 \\ 18{,}667 \end{bmatrix} = \mathrm{B}^{\infty} \begin{bmatrix} 809 \\ 1{,}255 \\ 18{,}365 \end{bmatrix}$$

This translates into a reduction of 26 percent in atmospheric concentration and a rise of 1.6% in deep ocean concentration. If the entire estimated amount of fossil fuels is spewed out into the atmosphere, over the very long run, the atmospheric concentration would stabilize at 713 GTC, lower than today's level, but in the intermediate years, concentration levels could rise dramatically. In the DICE baseline with no mitigation efforts, concentration levels in the atmosphere max out at around 3,000 GTC in around 2250.

The matrix B is valid for a time horizon spanning 10 years. In other words, equation (C-14) in terms of the DICE model is:

$$Conc_{z,t+10} = \mathrm{B}C_{z,t} + E_t$$

where E_t represents the cumulated emissions over 10 years through year t. It is possible to convert B into an annual transition matrix with some matrix algebra and numerical evaluation. If the matrix B is a positive definite matrix, than all of its eigenvalues are positive and it is possible to take the n^{th} root of the matrix B. The eigenvalues and eigenvectors of a real matrix B solve the following matrix equation:

$$\mathrm{B}x = \lambda x$$

In other words the projection of the vector x, by the matrix B is equal to that same vector multiplied by a scalar, λ. The eigenvalues, λ, can be calculated by solving an n-degree polynomial derived from the determinant of the above system:

$$\mathrm{B}x = \lambda x \Leftrightarrow (\mathrm{B} - \lambda.I)x = 0 \Rightarrow |\mathrm{B} - \lambda.I| = 0$$

Let V be the matrix of (right) eigenvectors of B (in columns), and Λ the diagonal matrix composed of the eigenvalues (in the same order as the respective eigenvectors), then B is diagonalized by:

$$\mathrm{B} = V\Lambda V^{-1}$$

It can be shown that if Λ has only positive eigenvalues[59], than the n^{th} root of B can be derived from[60]:

[59] If the diagonal elements of a square matrix B are all positive, and if B and B' are both diagonally dominant, then B is positive definite. The definition of diagonally dominant is that the absolute value of each diagonal element is greater than the sum of absolute values of the non-diagonal elements in its row. That is if for all i $|a(i,i)| >$ Sum($|a(i,j)|$;j != i).

[60] It is pretty easy to see this if n=2:

$$B^{1/n} = V\Lambda^{1/n}V^{-1}$$

In the case of the B matrix above, a numerical package has been used to numerically calculate the eigenvalues and eigenvectors[61]:

$$\Lambda = \begin{bmatrix} 0.694258 & 0 & 0 \\ 0 & 0.966122 & 0 \\ 0 & 0 & 1.000000 \end{bmatrix} \qquad V = \begin{bmatrix} -0.635745 & -0.311457 & 0.031954 \\ 0.761574 & -0.497912 & 0.062219 \\ -0.125829 & 0.809369 & 0.997551 \end{bmatrix}$$

Thus the annual transition matrix is given by:

$$K = B^{1/10} = V\Lambda^{1/10}V^{-1} = \begin{bmatrix} 0.978025 & 0.011566 & -0.000017 \\ 0.022520 & 0.983021 & 0.000338 \\ -0.000545 & 0.005413 & 0.999680 \end{bmatrix}$$

Intuitively, one can see that the diagonal elements of K are roughly equal to the diagonal elements of B raised to the power 0.1 and that the off-diagonal elements are roughly 10% of the off-diagonal elements of B.

[N.B. The third eigenvector reflects the same distribution as the long-run equilibrium distribution described in note 1 above, corresponding to the eigenvalue 1. The equilibrium matrix can also be derived from the following formula:

$$B^{\infty} = \underset{n\to\infty}{Lim}\, VK^{n}V^{-1} = V\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}V^{-1} = \begin{Bmatrix} 0.029269 & 0.029269 & 0.029269 \\ 0.056991 & 0.056991 & 0.056991 \\ 0.913739 & 0.913739 & 0.913739 \end{Bmatrix}$$

]

Equation (C-4) can then be written in cumulative form as:

$$Conc_t = K^n Conc_{t-n} + \sum_{j=0}^{n-1} K^{n-1-j} E_{t+j-n}$$

Assuming that emissions grow at a compound growth rate of g^e between *t-n* and *t*, we have the following:

$$C = V\Lambda^{1/2}V^{-1} = B^{1/2} \Leftrightarrow C.C = B \Leftrightarrow V\Lambda^{1/2}V^{-1}.V\Lambda^{1/2}V^{-1} = V\Lambda^{1/2}.\Lambda^{1/2}V^{-1} = B$$

In the next to the last step the square root of the diagonal matrix is simply the square root of each diagonal element and the multiplication of the two diagonal matrices is simply the original diagonal matrix. This is easy to generalize for any integer root.

61 The eigenvectors are determined up to a scalar multiple. In the case above, they have been normalized to be on the unit circle.

$$
\begin{aligned}
Conc_t &= \mathrm{K}^n Conc_{t-n} + \sum_{j=0}^{n-1} \mathrm{K}^{n-1-j}(1+g)^j E_{t-j-1} \\
&= \mathrm{K}^n Conc_{t-n} + (1+g)^{n-1} \sum_{j=0}^{n-1} V\Lambda^{n-1-j}(1+g)^{-n+1+j} V^{-1} E_{t-n} \\
&= \mathrm{K}^n Conc_{t-n} + (1+g)^{n-1} V \left[\sum_{j=0}^{n-1} \Lambda^{n-1-j}(1+g)^{-n+1+j} \right] V^{-1} E_{t-n}
\end{aligned}
$$

The expression within brackets is a diagonal matrix, so it is possible to use standard formulas for a geometric progression to give the following:

$$
Conc_t = V\Lambda^n V^{-1} Conc_{t-n} + V\Phi V^{-1} E_{t-n}
$$

where Λ is defined as above, and the diagonal matrix Φ is given by:

$$
\Phi_{ij} = \begin{cases} \dfrac{\lambda_i^n - (1+g^e)^n}{\lambda_i - (1+g^e)} & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}
$$

Based on these expressions, equation (1) that defines concentration for a one period gap is replaced by equations (4), (5) and (6). Equation (4) determines the growth of emissions between period *t-n* and *t* assuming a constant annual growth rate, g^{emi}. Equation (5), similar to the expression above, defines a 3 x 3 matrix, *EMIGFact*, which is used to determine the growth factor in the cumulative concentration expression. The parameter λ^c in the expression represents the eigenvalues of the transition matrix K. And equation (6), replacing equation (1) determines the cumulative concentration, *Conc*, in period *t*. The first component on the right is the evolution of the existing stock of carbon concentration where V^c is the matrix of eigenvectors of the K matrix and Λ^c is the diagonal matrix of eigenvalues. The second component represents cumulative emissions over the period *n*, with adjustments for the transition of the lagged emissions across the sinks.

(4) $$g^{emi}_{em,t} = \left(\frac{EMIGbl_{Armos,em,t}}{EMIGbl_{Armos,em,t-n}} \right)^{1/n} - 1$$

(5) $$EMIGFact_{z,t} = \frac{(\lambda_z^c)^n - (1+g^{emi}_{em,t})^n}{\lambda_z^c - (1+g^{emi}_{em,t})}$$

(6) $$Conc_t = V^c (\Lambda^c)^n V^{c^{-1}} Conc_{t-n} + (12/44).V^c EMIGFact_t V^{c^{-1}} EMIGbl_{z,CO2,t-n}$$

Temperature

The temperature transition, similar to concentration, has to be modified to allow for multiple year gaps. This section describes how the DICE formulation has been adapted for variable time steps.

The temperature module in DICE can be collapsed into matrix form:

$$T = M.T_{-1} + \mathrm{B}.F = \begin{bmatrix} 1-\beta_1(\lambda+\beta_2) & \beta_1\beta_2 \\ \beta_3 & 1-\beta_3 \end{bmatrix}.T_{-1} + \begin{bmatrix} \beta_1 \\ 0 \end{bmatrix}.F$$

where the transition and impact matrices, M and B are defined for a 10-year transition period. In the steady-state, this can be written as:

$$T^e = [I - M]^{-1} B.F$$

where F is a constant level of radiative forcing. The inverse matrix has a rather simple expression:

$$[I - M]^{-1} = \begin{bmatrix} 1/(\lambda\beta_1) & \beta_2/(\lambda\beta_1) \\ 1/(\lambda\beta_1) & (\lambda+\beta_2)/(\lambda\beta_3) \end{bmatrix}$$

This implies that the equilibrium temperature for both the atmosphere and the deep ocean is given simply by:

$$T^e = F/\lambda$$

With the default value for λ, the equilibrium temperature is about 0.8 times the equilibrium forcing level.

Similar to the concentration equation above, the temperature equation is recursive and can be collapsed into multi-period form by the following formula:

$$T_{t+n} = V\Lambda^n V^{-1} T_t + V\Phi V^{-1}\mathrm{B}.F_{t+n}$$

Where Λ is the diagonal matrix of eigenvalues of the one-period transition matrix, and the diagonal matrix Φ is given by:

$$\Phi_{ij} = \begin{cases} \dfrac{\lambda_i^n - (1+g^f)^n}{\lambda_i(1+g^f)^{n-1} - (1+g^f)^n} & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

where g^f is the annual compound growth rate of forcing (F) over the n-period range.

There are two differences with the concentration accumulation equation. The first is that the forcing variable, *F*, is pre-multiplied by the matrix B. The second is that in the DICE code the forcing variable is contemporaneous and not lagged—this changes the accumulation expression compared to the one for concentration. Both nevertheless collapse to 1 when *n* is equal to 1.

Similar to the concentration matrix, but with additional complications, one must convert the DICE-based 10-period M and B matrices into a 1-period matrix—hopefully preserving as well the particular relations across the different cells of the matrices. The following steps provide one way to do this:

1. Calculate the 10-period eigenvalues and eigenvectors of the 10-period M matrix so that the following holds:

$$M = V\Lambda V^{-1}$$

2. Calculate the 1/10th roots of the eigenvalues and then evaluate the 1-period M matrix, Γ:

$$\Gamma = V\Lambda^{0.1}V^{-1}$$

3. Calculate the β coefficients consistent with the values of the cells in Γ. There are too few degrees of freedom, so some choices must be made. For example:

$$\beta_2 = \frac{\lambda\Gamma_{12}}{1-\Gamma_{11}-\Gamma_{12}} \qquad \beta_1 = \frac{\Gamma_{12}}{\beta_2} \qquad \beta_3 = \Gamma_{21} \qquad \Gamma_{22} = 1-\beta_3$$

Thus, the bottom right cell of Γ is adjusted so that the sum along the bottom row is 1. The one-period B matrix, B_1 then becomes:

$$B_1 = \beta_1$$

4. Since the Γ matrix has been modified, it is necessary to re-calculate the eigenvalues and eigenvectors consistent with the adjusted 1-period Γ matrix. The new one-period β coefficients and the one-period eigenvalues and eigenvectors can be used for models that use one- or multi-period steps.

Equations (2) and (3) are then replaced with (7) and (8), (9) and (10). In equation (7) the only difference with Equation (2) is that the expression uses the average concentration between years *t-n* and *t*, rather than the concentration of a single year. Equation (8) defines the average annual growth rate in forcing, g^f, between years *t-n* and *t*. Equation (9) defines a 2 x 2 diagonal matrix that is used to provide the forcing growth factor, *ForcGFact*, for the cumulative temperature transition equation. It is similar to expression (5) save that the denominator is adjusted to account for the fact that forcing is assumed to impact current temperatures, and not future temperatures, i.e. forcing and temperature are contemporaneous variables. The λ^t parameters are the eigenvalues of the temperature transition matrix. Equation (10) represents the temperature/forcing relation for a multi-year transition period, where V^t is the matrix of eigenvectors of the temperature transition matrix, Λ^t is the matrix of eigenvalues, and Θ is the direct impact of forcing on temperature.

(7)	$Forc_{atmos} = fCO2x.\frac{\log_{10}\left(0.5(Conc_{atmos,t-n} + Conc_{atmos,t})/ConcPI\right)}{\log_{10}(2)} + ForcOth$
(8)	$g_t^f = \left(\frac{Forc_t}{Forc_{t-n}}\right)^n - 1$
(9)	$ForcGFact_{zt,t} = \frac{(\lambda_{zt}^t)^n - (1+g_t^f)^n}{\lambda_{zt}^t(1+g_t^f)^{n-1} - (1+g_t^f)}$
(10)	$Temp_{zt,t} = V^t(\Lambda^t)^n V^{t^{-1}} Temp_{zt,t-n} + V^t ForcGFact_t V^{t^{-1}} \Theta.Forc_{zt}$

Annex 7: Examples of carbon taxes, emission caps and tradable permits

Border tariff adjustment simulations

Border tariff adjustments (BTAs) have been a hotly contested subject since at least the early negotiations over the Kyoto Protocol. The issue arises when a small coalition of countries unilaterally implement emission reduction policies that raise domestic energy prices but have no direct impact on energy prices outside the coalition. This has two consequences. The first is that the coalition countries witness a loss in competitiveness and therefore market share—both at home and on export markets. And, their emission reduction efforts could be partially—or even entirely—offset by rising emission in non-coalition countries, the so-called carbon leakage effect. One response to either or both of these effects is to raise a carbon tax on competing imports thereby raising the relative price of imports with the aim of neutralizing the competiveness effect without of course affecting directly market shares in other countries. The coalition countries are also likely to rebate the carbon tax to exporters to maintain competitiveness outside the coalition. If the main concern is competitiveness, coalition countries could use the domestic carbon content to calculate the needed border tax adjustment. On the other hand, if the main concern is leakage, there could be a case for using the carbon content of the import competing countries under an assumption that all producers would therefore be paying the same price for their carbon emissions. This section describes how the ENVISAGE model has been modified to handle BTAs under these different assumptions.

Adjustment based on carbon content of domestic producers (or importers)

The main idea is to level the playing field for domestic producers. The model calculates how much the costs of production increase due to both the direct and indirect effects of the carbon tax. It is an ex ante calculation in the sense that the cost structure of a reference year is chosen before implementation of the BTA and assumes a given price and production structure. In a GE framework prices and quantities will adjust so that the ex post increase in cost is likely to be lower than the ex ante increase thus the BTA will over-compensate domestic producers.

Equation (A7-1) defines the ex ante increase in unit cost, PX^1, generated by the carbon tax, where the year *tr*, refers to a reference year prior to the implementation of the BTA and the super-scripted price variables are the prices that reflect the direct and indirect costs of the carbon tax. It assumes therefore a fixed technology and the same prices for factors—but augments the cost of intermediate goods by the carbon tax. The new unit cost will include both the direct and indirect costs of the carbon tax. Equation (A7-2) determines the ex ante price of intermediate goods, PA^1. It is assumed that the increase in intermediate goods is equal to the increase in the unit cost. Equations (A7-1) and (A7-2) represent a recursive system in prices that will generate in the end the relative increase in the unit cost of production induced by carbon taxation. Equation (A7-3) calculates the ex-ante wedge in the unit cost, ω^d, i.e. the wedge that is induced by the carbon tax given the cost structure of the reference year. Equation (A7-4) calculates the equivalent tariff that is applied to imported goods that offsets the ex ante increase in the cost of domestic production. A complicating factor is the multi-output structure of production. This is dealt with by taking the weighted average of the different production streams (indexed by *a*) to produce commodity, *i*. The weights are represented by the α^p parameter and are calculated using the reference year shares:

$$\alpha^p_{r,a} = \frac{\sum_{a \in \gamma^p_{r,a,i} \neq 0} PP_{r,a,tr} XP_{r,a,tr}}{\sum_{a \in \gamma^p_{r,a,i} \neq 0} PS_{r,i,tr} XS_{r,i,tr}}$$

For most commodity/activity combinations there is a one-to-one correspondence and the α^p parameter takes the value 1. Note that equations (A7-1) and (A7-2) hold for all activities and commodities, whereas equation (A7-4) is only applied to a subset of commodities indexed by *it*. The index *r* represents countries that are self-imposing a carbon tax, whereas the index *r*' in equation (A7-4) is for all countries that are not limiting emissions. Thus, even if there is uneven effort (or uneven carbon taxes) across countries with GHG emission limits, there is no assumption that a compensating mechanism will be in effect among these countries.

(A7-1) $$PX^1_{r,a,t} = \frac{\sum_i PA^1_{r,i,a,t} XA_{r,i,a,tr} + \sum_{fp} PF_{r,fp,a,tr} XF_{r,fp,a,tr} + \sum_i \sum_{em} \tau^{emi}_{r,em,t} \rho_{r,em,i,a} XA_{r,i,a,tr}}{XP_{r,a,tr}}$$

(A7-2) $$PA^1_{r,i,a,t} = PA_{r,i,tr} \frac{\sum_{a' \in \gamma^p_{r,a,i} \neq 0} \alpha^p_{r,a'} PX^1_{r,a',t}}{\sum_{a' \in \gamma^p_{r,a,i} \neq 0} \alpha^p_{r,a'} PX_{r,a',tr}}$$

(A7-3) $$\omega^d_{r,a,t} = \frac{PX^1_{r,a,t} - PX_{r,a,tr}}{PX_{r,a,tr}}$$

(A7-4) $$\tau^a_{r',r,it,t} = \sum_{a \in \gamma^p_{r,a,it} \neq 0} \alpha^p_{r,a} \omega^d_{r,a,t}$$

Most border adjustment regimes would also include a cost-compensation for exports. A symmetric additive adjustment factor can be included to existing export taxes subsidies. Equation (A7-5) defines the export subsidy adjustment, where *r* is the exporting country and has imposed a carbon tax, and *r*' is a destination country with no carbon tax.

(A7-5) $$\tau^{ae}_{r,r',it,t} = -\sum_{a \in \gamma^p_{r,a,it} \neq 0} \alpha^p_{r,a} \omega^d_{r,a,t}$$

These new instruments require changes to equations that contain bilateral tariffs and/or export tax subsidies. This reduces to four equations—equations (T-7) and (T-9) that define bilateral trade prices, FOB and end-user respectively; and equations (Y-3) and (Y-4) that determine fiscal revenues associated with import tariffs and export subsidies respectively.

(T-7') $$WPE_{r,r',im} = (1+\tau^{e}_{r,r',im})PE_{r,r',im}$$

(T-9') $$PM_{r,r',im} = (1+\tau^{m}_{r,r',im})WPM_{r,r',im}$$

(Y-3') $$GREV_{r,ttax} = \sum_{i \in Arm} \sum_{r'} \left(\tau^{m}_{r',r,i} + \tau^{a}_{r',r,i}\right) WPM_{r',r,i} WTF^{d}_{r',r,i} + \sum_{i \notin Arm} \tau^{m}_{r,i} PW_i XMT_{r,i}$$

(Y-4') $$GREV_{r,etax} = \sum_{i \in Arm} \sum_{r'} \left(\tau^{e}_{r,r',i} + \tau^{ae}_{r,r',i}\right) PE_{r,r',i} WTF^{s}_{r,r',i} + \sum_{i \notin Arm} \tau^{e}_{r,i} PW_i XET_{r,i}$$

Adjustment based on carbon content of exporters

A tax on the carbon content of imports may help to attenuate the leakage effect of unilateral carbon taxes. The implementation involves assessing the ex ante cost of production in the exporting countries (i.e. those not imposing a carbon tax) applying the carbon tax of the destination country. This implies that the cost wedge and the tariff adjustment will vary across exporting countries for the same destination economy, unlike the adjustment above where the tariff adjustment is uniform across importing countries.

Equation (A7-6) defines the *fictitious* carbon tax that is imposed on the cost of production in the exporting country. It is bilateral as each of the importing countries has a different carbon tax. Equation (A7-7) defines the ex ante increase in unit cost, PX^2, generated by the carbon tax, where the year *tr*, refers to a reference year prior to the implementation of the BTA and the super-scripted price variables are the prices that reflect the direct and indirect costs of the carbon tax. It assumes therefore a fixed technology and the same prices for factors—but augments the cost of intermediate goods by the carbon tax. The new unit cost will include both the direct and indirect costs of the carbon tax. It is a bilateral price as the carbon tax differs across countries of destination. Equation (A7-8) determines the ex ante price of intermediate goods, PA^2. It is assumed that the increase in intermediate goods is equal to the increase in the unit cost. Equation (A7-9) calculates the ex-ante wedge in the unit cost, ω^m, i.e. the wedge that is induced by the carbon tax given the cost structure of the reference year. Equation (A7-10) calculates the equivalent tariff that is applied to imported goods that offsets the ex ante increase in the cost of domestic production. Note that equations (A7-7) through (A7-9) hold for all activities and commodities, whereas equation (A7-10) is only applied to a subset of commodities indexed by *it*. The index *r*' represents countries that are self-imposing a carbon tax, whereas the index *r* is for all countries that are not limiting emissions.

(A7-6) $\tau^{emi,2}_{r',r,em,t} = \tau^{emi}_{r',em,t}$

(A7-7) $PX^{2}_{r',r,a,t} = \dfrac{\sum_{i} PA^{2}_{r',r,i,a,t} XA_{r,i,a,tr} + \sum_{fp} PF_{r,fp,a,tr} XF_{r,fp,a,tr} + \sum_{i}\sum_{em} \tau^{emi,2}_{r',r,em,t} \rho_{r,em,i,a} XA_{r,i,a,tr}}{XP_{r,a,tr}}$

(A7-8) $PA^{2}_{r',r,i,a,t} = PA_{r,i,tr} \dfrac{\sum_{a' \in \gamma^{p}_{r,a,i} \neq 0} \alpha^{p}_{r,a'} PX^{2}_{r',r,a',t}}{\sum_{a' \in \gamma^{p}_{r,a,i} \neq 0} \alpha^{p}_{r,a'} PX_{r,a',tr}}$

(A7-9) $\omega^{m}_{r',r,a,t} = \dfrac{PX^{2}_{r',r,a,t} - PX_{r,a,tr}}{PX_{r,a,tr}}$

(A7-10) $\tau^{a}_{r,r',it,t} = \sum_{a \in \gamma^{p}_{r,a,it} \neq 0} \alpha^{p}_{r,a} \omega^{d}_{r',r,a,t}$

Some scenarios allow for compensating a subset of regions for the imposition of a carbon tax. Thus while high-income countries benefit from a uniform global price on carbon, developing countries may not desire to impose a tax that could be harmful to their economy, especially since high-income countries are as at present mostly responsible for the current stock of GHG atmospheric concentration. The compensation mechanism is implemented as a government to government transfer, *GTR*. Given the fiscal closure rule, this implies that direct taxes on households will shift—lower taxes for receiving countries and higher taxes for donor countries. Equation (A7-11) guarantees that the sum of the transfers adds up to zero globally where countries are divided into donor and recipient countries. (N.B. The transfers are not bilateral.) Equation (A7-12) is an allocation mechanism across donor countries that insures that the transfers are equalized on a per capita basis.

(A7-11) $\sum_{r \in Donors} GTR_r + \sum_{r \in Recipients} GTR_r \equiv 0$

(A7-12) $GTRpc = GTR_r / Pop_r \quad \text{for} \quad r \in Donors$

The implementation requires exogenizing some objective for the recipient countries. In the standard implementation, real domestic absorption is fixed to baseline values, where the superscript *BaU* refers to the baseline level:

$$RYD_{r,t} = RYD^{BaU}_{r,t} \quad \text{for} \quad r \in Recipients$$

The addition of the new *GTR* variable requires a change to the government revenue equation:

(Y-8') $YG_r = \sum_{gy} GREV_{r,gy} + \sum_{em} QuotaY^{E}_{em} + GTR_r$

Annex 8: Base ENVISAGE parameters

Like most CGE models, ENVISAGE contains a mix of calibrated and key parameters—the latter sourced from a variety of studies. The basic framework of a comparative static CGE model is that a base year data set is given—the GTAP world social accounting matrix (SAM), for example. This represents value flows. Base prices are typically initialized at unit value, with some exceptions if volume flows and/or stocks are available—for example energy in physical units or the stock of labor. Parameters are then divided into two sets: key parameters—typically substitution, supply, price and income elasticities, and calibrated parameters. The model can be represented compactly by the following formula:

$$F(Y, X; \theta_1, \theta_2) = 0$$

where Y represents endogenous variables, X is the set of exogenous variables including policy instruments, θ_1 is the set of key parameters and θ_2 is the set of calibrated parameters. The key parameters are given and typically estimated using outside sources and data. In the calibration phase, both Y and X are given (by the base year data), and the function F is inverted to calibrate the θ_2 parameters such that the model can replicate the base year data.[62] Alternative scenarios then involve perturbing one or more elements in X and inverting the function F to calculate a new Y, the set of endogenous variables, holding θ_1 and θ_2 constant. Note that sensitivity analysis on θ_1, the key parameters, typically requires re-calibrating θ_2 for each new set of θ_1. The rest of this annex presents the key parameter values used for ENVISAGE.

Production elasticities

The basic production substitution elasticities are provided in Tables A8.1 and A8.2. They replicate those used for the OECD GREEN model (see Burniaux et al 1992) and likewise underlie the World Bank's LINKAGE model. The original OECD GREEN model had a single energy nest rather than the nested structure of ENVISAGE. The uniformity of the energy substitution elasticities therefore replicates the structure of GREEN.

Table A8.1 Production parameters

Name	Symbol	Old	New	Note
`sigmap`	σ^p	0.00	0.00	
`sigmav`	σ^v	0.12	1.00	Leontief technology in fossil fuel production
`sigmake`	σ^{ke}	0.00	0.80	Leontief technology in fossil fuel production
`sigman`	σ^n	0.00	0.00	Not differentiated by vintage

62 Invariably, calibration can be done block by block and does not involve actually inverting F as a complete system of equations.

Table A8.2 Energy substitution elasticities in production

Name	Symbol	Old	New	Note
sigmae	σ^{e}	0.25	2.00	
sigmanely	σ^{nely}	0.25	2.00	
sigmaolg	σ^{olg}	0.25	2.00	
sigmaely	σ^{ely}	0.25	2.00	
sigmacoa	σ^{coa}	0.25	2.00	
sigmaoil	σ^{oil}	0.25	2.00	
sigmagas	σ^{gas}	0.25	2.00	

Final demand elasticities

Consumer final demand elasticities in the ENVISAGE models are derived in large part from estimates produced by the Economic Research Service (ERS) of the U.S. Department of Agriculture (USDA).[63] The available estimates are for a reduced set of goods and these estimates are allocated over the 57 sectoring scheme of GTAP. They are aggregated using GTAP consumption shares for specific aggregations of GTAP. Table A8.3 provides the income elasticities for all 57 GTAP goods for a selected aggregation across GTAP regions.[64] The calibration procedure of ENVISAGE may make some adjustments to these elasticities to insure that the consumption weighted sum of the income elasticities adds up to unity. Table A8.4 provides the initial price elasticities used in the calibration procedure for the same aggregate regions as in Table A8.3—again using simple averages within regions rather than consumption weighted.

Equations (D-12) through (D-13) convert consumed goods to produced goods using a transition matrix approach with a CES preference structure. The transition matrix is currently diagonal

63 Regmi 2001, Seale et al 2003 and Regmi and Seale 2010.

64 Regional aggregations in Table A8.3 are simple—not consumption weighted.

Table A8.3 Consumer income elasticities for select regions

	CHN	XEA	IND	XSA	RUS	XEC	MNA	SSA	LAC	WEU	JPN	USA	RHY
PDR	0.19	0.46	0.49	0.49	0.40	0.41	0.43	0.52	0.38	0.22	0.16	0.05	0.18
WHT	0.19	0.46	0.48	0.49	0.40	0.40	0.42	0.52	0.38	0.22	0.16	0.05	0.18
GRO	0.19	0.46	0.49	0.49	0.40	0.40	0.42	0.43	0.38	0.22	0.16	0.05	0.18
V_F	0.38	0.55	0.58	0.55	0.50	0.49	0.51	0.50	0.48	0.33	0.24	0.08	0.29
OSD	0.23	0.48	0.50	0.52	0.43	0.43	0.45	0.54	0.41	0.25	0.18	0.06	0.21
C_B	0.48	0.74	0.76	0.77	0.65	0.66	0.68	0.76	0.64	0.43	0.31	0.11	0.39
PFB	0.91	0.92	0.92	0.92	0.91	0.91	0.91	0.92	0.91	0.91	0.90	0.90	0.91
OCR	0.48	0.73	0.74	0.75	0.65	0.66	0.68	0.68	0.63	0.43	0.31	0.11	0.38
CTL	0.48	0.74	0.77	0.74	0.66	0.65	0.68	0.74	0.64	0.43	0.31	0.11	0.39
OAP	0.48	0.70	0.75	0.74	0.65	0.65	0.68	0.74	0.64	0.43	0.31	0.11	0.39
RMK	0.51	0.81	0.84	0.76	0.68	0.71	0.74	0.84	0.69	0.46	0.33	0.12	0.41
WOL	0.91	0.92	0.92	0.92	0.91	0.91	0.92	0.92	0.91	0.91	0.90	0.90	0.91
FRS	1.26	1.47	1.49	1.47	1.33	1.37	1.36	1.70	1.34	1.26	1.24	1.22	1.25
FSH	0.53	0.79	0.87	0.84	0.74	0.75	0.77	0.87	0.72	0.48	0.34	0.12	0.42
COA	1.17	1.23	1.24	1.24	1.19	1.21	1.20	1.27	1.20	1.17	1.16	1.15	1.16
OIL	1.18	1.25	1.26	1.26	1.21	1.22	1.22	1.29	1.21	1.18	1.17	1.16	1.17
GAS	1.17	1.23	1.24	1.24	1.19	1.20	1.20	1.27	1.20	1.17	1.16	1.15	1.16
OMN	1.26	1.48	1.50	1.49	1.33	1.37	1.36	1.74	1.35	1.26	1.24	1.22	1.25
CMT	0.48	0.73	0.77	0.77	0.61	0.65	0.67	0.74	0.60	0.42	0.31	0.10	0.38
OMT	0.48	0.70	0.77	0.77	0.64	0.65	0.64	0.72	0.61	0.41	0.31	0.10	0.38
VOL	0.24	0.46	0.51	0.48	0.42	0.43	0.45	0.51	0.39	0.25	0.18	0.06	0.20
MIL	0.51	0.79	0.82	0.82	0.69	0.69	0.71	0.83	0.66	0.43	0.33	0.11	0.41
PCR	0.19	0.39	0.46	0.38	0.40	0.41	0.43	0.49	0.36	0.22	0.15	0.05	0.17
SGR	0.48	0.73	0.76	0.73	0.64	0.66	0.67	0.74	0.63	0.43	0.31	0.11	0.38
OFD	0.48	0.61	0.72	0.73	0.61	0.62	0.64	0.58	0.57	0.39	0.27	0.08	0.36
B_T	0.58	0.90	1.07	1.03	0.85	0.87	0.90	1.18	0.79	0.51	0.36	0.12	0.45
TEX	0.91	0.87	0.88	0.86	0.91	0.90	0.89	0.88	0.89	0.89	0.90	0.90	0.90
WAP	0.91	0.84	0.92	0.90	0.89	0.90	0.87	0.88	0.86	0.88	0.89	0.88	0.88
LEA	0.91	0.89	0.92	0.92	0.91	0.91	0.91	0.91	0.89	0.90	0.90	0.90	0.90
LUM	1.26	1.47	1.50	1.49	1.32	1.36	1.35	1.73	1.33	1.26	1.24	1.22	1.25
PPP	1.26	1.46	1.49	1.49	1.33	1.36	1.35	1.73	1.32	1.24	1.23	1.21	1.24
P_C	1.18	1.15	1.20	1.24	1.17	1.19	1.17	1.25	1.14	1.14	1.15	1.14	1.15
CRP	1.27	1.41	1.48	1.45	1.30	1.34	1.33	1.67	1.26	1.23	1.23	1.20	1.23
NMM	1.26	1.47	1.50	1.48	1.33	1.37	1.35	1.73	1.34	1.26	1.24	1.22	1.25
I_S	1.26	1.48	1.50	1.49	1.33	1.37	1.36	1.74	1.34	1.26	1.24	1.22	1.25
NFM	1.26	1.47	1.50	1.49	1.33	1.37	1.36	1.74	1.34	1.26	1.24	1.22	1.25
FMP	1.26	1.47	1.49	1.49	1.33	1.37	1.36	1.73	1.34	1.26	1.24	1.22	1.25
MVH	1.18	1.17	1.26	1.25	1.18	1.20	1.18	1.27	1.18	1.13	1.15	1.13	1.14
OTN	1.18	1.23	1.26	1.26	1.21	1.22	1.21	1.29	1.19	1.17	1.17	1.15	1.17
ELE	1.27	1.41	1.49	1.48	1.33	1.36	1.35	1.73	1.31	1.25	1.22	1.21	1.22
OME	1.27	1.43	1.49	1.49	1.31	1.34	1.35	1.73	1.32	1.24	1.23	1.20	1.23
OMF	1.26	1.41	1.49	1.48	1.32	1.36	1.34	1.71	1.30	1.24	1.23	1.21	1.23
ELY	1.17	1.17	1.22	1.21	1.12	1.13	1.16	1.24	1.16	1.13	1.15	1.14	1.15
GDT	1.17	1.23	1.24	1.24	1.18	1.17	1.20	1.27	1.20	1.17	1.16	1.15	1.16
WTR	1.17	1.23	1.24	1.24	1.18	1.20	1.20	1.25	1.19	1.16	1.15	1.14	1.16
CNS	1.17	1.23	1.24	1.24	1.19	1.20	1.20	1.26	1.19	1.16	1.16	1.15	1.16
TRD	1.19	0.92	1.17	1.19	0.97	1.11	1.08	1.08	1.03	1.01	0.99	0.98	0.94
OTP	1.18	1.19	1.22	1.15	1.15	1.18	1.18	1.23	1.12	1.14	1.13	1.14	1.14
WTP	1.18	1.25	1.25	1.23	1.20	1.22	1.22	1.29	1.21	1.17	1.17	1.16	1.17
ATP	1.18	1.22	1.26	1.23	1.20	1.21	1.21	1.28	1.19	1.17	1.16	1.16	1.15
CMN	1.18	1.22	1.26	1.25	1.20	1.20	1.20	1.27	1.17	1.15	1.15	1.13	1.15
OFI	1.27	1.44	1.49	1.49	1.33	1.36	1.34	1.70	1.30	1.23	1.23	1.15	1.22
ISR	1.26	1.45	1.50	1.48	1.33	1.37	1.35	1.72	1.32	1.24	1.21	1.18	1.22
OBS	1.27	1.41	1.48	1.46	1.28	1.33	1.27	1.67	1.31	1.09	1.22	1.21	1.22
ROS	1.33	1.71	1.79	1.71	1.42	1.52	1.47	2.47	1.41	1.28	1.23	1.16	1.25
OSG	1.28	1.42	1.49	1.46	1.30	1.36	1.29	1.77	1.25	1.19	1.18	0.99	1.16
DWE	1.18	1.06	1.18	1.18	1.20	1.19	1.17	1.21	1.02	1.16	1.02	0.99	1.03

Note: The regional abbreviations are China (CHN), Rest of East Asia & Pacific (XEA), India (IND), Rest of South Asia (XSA), Russia (RUS), Rest of Europe & Central Asia (XEC), Middle East & North Africa (MNA), Sub-Saharan Africa (SSA), Latin America & Caribbean (LAC), Western Europe (WEU), Japan (JPN), the United States (USA), and Rest of High-income (RHY).

Table A8.4 Consumer price elasticities for select regions

	CHN	XEA	IND	XSA	RUS	XEC	MNA	SSA	LAC	WEU	JPN	USA	RHY
PDR	-0.15	-0.37	-0.41	-0.40	-0.33	-0.33	-0.35	-0.43	-0.31	-0.18	-0.13	-0.04	-0.14
WHT	-0.15	-0.37	-0.41	-0.40	-0.33	-0.33	-0.35	-0.43	-0.31	-0.18	-0.13	-0.04	-0.14
GRO	-0.15	-0.37	-0.41	-0.40	-0.33	-0.33	-0.35	-0.43	-0.31	-0.18	-0.13	-0.04	-0.14
V_F	-0.30	-0.48	-0.51	-0.50	-0.43	-0.43	-0.44	-0.51	-0.42	-0.28	-0.20	-0.07	-0.25
OSD	-0.19	-0.39	-0.43	-0.42	-0.35	-0.35	-0.37	-0.44	-0.33	-0.20	-0.15	-0.05	-0.17
C_B	-0.39	-0.60	-0.62	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
PFB	-0.69	-0.69	-0.70	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69
OCR	-0.39	-0.60	-0.62	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
CTL	-0.39	-0.60	-0.63	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
OAP	-0.39	-0.60	-0.63	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
RMK	-0.41	-0.65	-0.68	-0.68	-0.58	-0.58	-0.60	-0.69	-0.56	-0.38	-0.27	-0.10	-0.34
WOL	-0.69	-0.69	-0.70	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69
FRS	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
FSH	-0.42	-0.68	-0.71	-0.71	-0.60	-0.61	-0.63	-0.73	-0.59	-0.39	-0.28	-0.10	-0.35
COA	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76
OIL	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
GAS	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76
OMN	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
CMT	-0.39	-0.60	-0.63	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
OMT	-0.39	-0.60	-0.63	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
VOL	-0.19	-0.39	-0.43	-0.42	-0.35	-0.35	-0.37	-0.44	-0.33	-0.20	-0.15	-0.05	-0.17
MIL	-0.41	-0.65	-0.68	-0.68	-0.58	-0.58	-0.60	-0.69	-0.56	-0.38	-0.27	-0.10	-0.34
PCR	-0.15	-0.37	-0.41	-0.40	-0.33	-0.33	-0.35	-0.43	-0.31	-0.18	-0.13	-0.04	-0.14
SGR	-0.39	-0.60	-0.62	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
OFD	-0.39	-0.60	-0.62	-0.62	-0.53	-0.54	-0.55	-0.62	-0.52	-0.35	-0.25	-0.09	-0.31
B_T	-0.47	-0.82	-0.88	-0.86	-0.71	-0.73	-0.76	-1.07	-0.69	-0.44	-0.31	-0.11	-0.38
TEX	-0.69	-0.69	-0.70	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69
WAP	-0.69	-0.69	-0.70	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69
LEA	-0.69	-0.69	-0.70	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69	-0.69
LUM	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
PPP	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
P_C	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
CRP	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
NMM	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
I_S	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
NFM	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
FMP	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
MVH	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
OTN	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
ELE	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
OME	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
OMF	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
ELY	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76
GDT	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76
WTR	-0.87	-0.93	-0.94	-0.94	-0.90	-0.91	-0.90	-0.96	-0.90	-0.87	-0.86	-0.85	-0.86
CNS	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76
TRD	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
OTP	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
WTP	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
ATP	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
CMN	-0.82	-0.91	-0.92	-0.92	-0.86	-0.87	-0.87	-0.95	-0.86	-0.82	-0.80	-0.79	-0.81
OFI	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
ISR	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
OBS	-0.86	-1.07	-1.09	-1.09	-0.94	-0.97	-0.96	-1.29	-0.95	-0.86	-0.83	-0.81	-0.85
ROS	-0.99	-1.41	-1.40	-1.39	-1.10	-1.17	-1.14	-1.98	-1.12	-0.99	-0.95	-0.93	-0.97
OSG	-0.91	-1.13	-1.15	-1.14	-0.98	-1.02	-1.01	-1.39	-1.00	-0.91	-0.88	-0.86	-0.90
DWE	-0.77	-0.85	-0.87	-0.87	-0.81	-0.82	-0.82	-0.89	-0.81	-0.77	-0.75	-0.74	-0.76

Note: The regional abbreviations are China (CHN), Rest of East Asia & Pacific (XEA), India (IND), Rest of South Asia (XSA), Russia (RUS), Rest of Europe & Central Asia (XEC), Middle East & North Africa (MNA), Sub-Saharan Africa (SSA), Latin America & Caribbean (LAC), Western Europe (WEU), Japan (JPN), the United States (USA), and Rest of High-income (RHY).

Annex 9: The accounting framework

This annex provides a visual representation of the accounting framework in the GTAP dataset and linked to the model specified in this document. We use the Social Accounting Matrix (or SAM) framework, which is somewhat space consuming, but has the advantage of providing a consistent picture in a single snapshot.[65] There is no unique representation of a SAM, the one depicted in Figure A8-2 has the advantage of reflecting the accounts in basic prices and with all individual price wedges.

The figure itself does not need much description. There are 18 accounts in total, some of which are purely pass-through accounts (to stick to the tradition that a SAM should be a square and balanced matrix). The following table summarizes the accounts:

Table A8-1: Description of the SAM accounts

Account	*Description*
ACT	Represents the production activities. The total is domestic output at producer price, the latter includes the producer tax. Revenues are exhausted by payments to intermediate goods (including sales tax) and to factors of production.
COMM	Represents total supply—domestic production and imports. The latter enter at CIF prices, to which are added import taxes. The disposition of total supply includes domestic sales of domestic goods, *XD*, aggregate imports, exports and supply of international trade and transport services.
DAP	This is the disposition of domestic sales of domestic production at producer price (*PD*).
MAP	This is the disposition of import sales at tariff inclusive import prices (that are uniform across all agents).
DIT	Revenues generated by the agent-specific sales tax on domestic products.
MIT	Revenues generated by the agent-specific sales tax on imported products.
VA	Value added accounts. In the activity column, it reflects the net of tax cost of the factors of production. All factor remuneration is attributed to the single representative household.
VA_TAX	Revenues from taxes on the factors of production. All tax revenues are attributed to the government account.
PTAX	Output tax revenues.
EXP_TAX	Revenues (or cost) from export taxes (or subsidies).
IMP_TAX	Revenues from import tariffs.
HH	Represents the accounts of the private sector. From a national account perspective, this is a consolidated private sector that includes enterprises and non-governmental organizations. In this SAM, the sole source of income for households is net factor remuneration. Expenditures include demand for goods and services and savings net of depreciation. Households save and pay income taxes to the government. Note that in the SAM database the fiscal accounts are not closed. The model is initialized to assume a zero government deficit and direct taxes represent a residual to balance the household (and government accounts).
GOV	The government collects all indirect taxes and purchases goods and services. Its account is closed by assuming a lump-sum tax on households.
INV	The investment account purchases goods and services. Its income comes from

[65] For an introduction to Social Accounting Matrices, see Pyatt and Round 1985 and Reinert and Roland-Holst 1997.

	domestic private savings gross of depreciation and foreign saving. Public saving is implicitly assumed to be zero.
DEPR	The depreciation account is a pass-through account.
TRADE	The trade account measures the flow of exports (by region of destination) at FOB prices and the flow of imports (by region of origin) at CIF prices. Aggregate exports and imports (across sectors) are recorded in the balance of payment accounts (BoP) by region. The total for these columns/rows is therefore the sum of exports and imports. The difference between exports and imports provides the net trade with each region (though using different prices since exports are evaluated FOB and imports are evaluated COF). Aggregate exports (by region) show up in the BoP column since they represent foreign income. Aggregate imports (by region) show up in the BoP row.
ITT_MARG	This account shows the regional supply of international trade and transport services. Its aggregate sum will show up in the BoP column since it is foreign revenue.
BoP	This account has the consolidated balance of payments. Exports and supply of international trade and transport services will show up as revenues in the column. Imports will show up in the row as a payment to the rest of the world. The balancing item is the capital account that appears in the column as a payment to the investment sector. If it is positive, the region is a net capital importer. If it is negative, the region is a net capital exporter. In the aggregation of all regional SAMs, this item should show up as a zero. Also, the sum of exports across all regions and the sum of international trade and transport services should equal the sum of imports.

Figure A8-2: The schematic Social Accounting Matrix

		ACT	COMM	DAP	MAP	DIT	MIT
		j	j	j	j	j	j
Activities	i		$Diag(PP_{r,j}XP_{r,j})$				
Produced commodities	i			$Diag(PD_{r,j}XDT_{r,j})$	$Diag(PMT_{r,j}XMT_{r,j})$		
Domestic commodities	i	$PD_{r,i}XD_{r,i,j}$					
Imported commodities	i	$PMT_{r,i}XM_{r,i,j}$					
Sales tax on dom. goods	i	$\tau^{Ad}_{r,i,j}PD_{r,i}XD_{r,i,j}$					
Sales tax on imp. goods	i	$\tau^{Am}_{r,i,j}PMT_{r,i}XM_{r,i,j}$					
Value added	fp	$NPF_{r,fp,j}XF_{r,fp,j}$					
Value added tax	fp	$\tau^{v}_{r,fp,j}NPF_{r,fp,j}XF_{r,fp,j}$					
Production tax	1	$\tau^{p}_{r,j}PX_{r,j}XP_{r,j}$					
Import tariffs	r'		$\tau^{m}_{r',r,j}WPM_{r',r,j}WTF_{r',r,j}$				
Export taxes	r'		$\tau^{e}_{r,r',j}PE_{r,r',j}WTF_{r,r',j}$				
Households	1						
Government	1					$\sum_a \tau^{Ad}_{r,j,a}PD_{r,j}XD_{r,j,a}$	$\sum_a \tau^{Am}_{r,j,a}PMT_{r,j}XM_{r,j,a}$
Investment	1						
Depreciation	1						
Trade	r'		$WPM_{r',r,j}WTF_{r',r,j}$				
International trade margins	1						
Balance of payments	1						

Figure A8-2: The schematic Social Accounting Matrix, continued

		VA	VA_TAX	PTAX	EXP_TAX	IMP_TAX	HH
		fp	fp	1	r'	r'	1
Activities	i						
Produced commodities	i						
Domestic commodities	i						$PD_{r,i}XD_{r,i,h}$
Imported commodities	i						$PMT_{r,i}XM_{r,i,h}$
Sales tax on dom. goods	i						$\tau^{Ad}_{r,i,h}PD_{r,i}XD_{r,i,h}$
Sales tax on imp. goods	i						$\tau^{Am}_{r,i,h}PMT_{r,i}XM_{r,i,h}$
Value added	fp						
Value added tax	fp						
Production tax	1						
Import tariffs	r'						
Export taxes	r'						
Households	1	$\sum_j NPF_{r,fp,j}XF_{r,fp,j}$					
Government	1		$\sum_j \tau^v_{r,fp,j}NPF_{r,fp,j}XF_{r,fp,j}$	$\sum_j \tau^p_{r,j}PX_{r,j}XP_{r,j}$	$\sum_i \tau^e_{r,r',i}PE_{r,r',i}WTF_{r,r',i}$	$\sum_i \tau^m_{r',r,i}WPM_{r',r,i}WTF_{r',r,i}$	$\chi^c_r \kappa^h_{r,h} YH_r$
Investment	1						$S^h_{r,h}$
Depreciation	1						$DEPRY_r$
Trade	r'						
International trade margins	1						
Balance of payments	1						

Figure A8-2: The schematic Social Accounting Matrix, continued

		GOV	INV	DEPR	TRADE	ITT_MARG	BOP
		1	1	1	r'	1	1
Activities	i						
Produced commodities	i				$WPE_{r,r',i}WTF_{r,r',i}$	$PP_{r,i}XMG_{r,i}$	
Domestic commodities	i	$PD_{r,i}XD_{r,i,Gov}$	$PD_{r,i}XD_{r,i,Inv}$				
Imported commodities	i	$PMT_{r,i}XM_{r,i,Gov}$	$PMT_{r,i}XM_{r,i,Inv}$				
Sales tax on dom. goods	i	$\tau^{Ad}_{r,i,Gov}PD_{r,i}XD_{r,i,Gov}$	$\tau^{Ad}_{r,i,Inv}PD_{r,i}XD_{r,i,Inv}$				
Sales tax on imp. goods	i	$\tau^{Am}_{r,i,Gov}PMT_{r,i}XM_{r,i,Gov}$	$\tau^{Am}_{r,i,Inv}PMT_{r,i}XM_{r,i,Inv}$				
Value added	fp						
Value added tax	fp						
Production tax	1						
Import tariffs	r'						
Export taxes	r'						
Households	1						
Government	1						
Investment	1	S^g_r		$DEPRY$			S^f_r
Depreciation	1						
Trade	r'						$\sum_i WPE_{r,r',i}WTF_{r,r',i}$
International trade margins	1						$\sum_i PP_{r,i}XMG_{r,i}$
Balance of payments	1				$\sum_i WPM_{r',r,j}WTF_{r',r,j}$		

Annex 10: Dimensions of GTAP release 7.1

Table A9-1: Regional Concordance

1	AUS	**Australia**
2	NZL	**New Zealand**
3	XOC	**Rest of Oceania** *American Samoa* (asm), *Cook Islands* (cok), *Fiji* (fji), *French Polynesia* (pyf), *Guam* (gum), *Kiribati* (kir), *Marshall Islands* (mhl), *Federated States of Micronesia* (fsm), *Nauru* (nau), *New Caledonia* (ncl), *Norfolk Island* (nfk), *Northern Mariana Islands* (mnp), *Niue* (niu), *Palau* (plw), *Papua New Guinea* (png), *Samoa* (wsm), *Solomon Islands* (slb), *Tokelau* (tkl), *Tonga* (ton), *Tuvalu* (tuv), *Vanuatu* (vut), *Wallis and Futura Islands* (wlf)
4	CHN	**China**
5	HKG	**Hong Kong (China)**
6	JPN	**Japan**
7	KOR	**Republic of Korea**
8	TWN	**Taiwan (China)**
9	XEA	**Rest of East Asia** *Macao* (mac), *Mongolia* (mng), *North Korea* (prk)
10	KHM	**Cambodia**
11	IDN	**Indonesia**
12	LAO	**Lao, PDR**
13	MYS	**Malaysia**
14	PHL	**Philippines**
15	SGP	**Singapore**
16	THA	**Thailand**
17	VNM	**Vietnam**
18	XSE	**Rest of Southeast Asia** *Brunei Darussalam* (brn), *Myanmar* (mmr), *Timor-Leste* (tmp)
19	BGD	**Bangladesh**
20	IND	**India**
21	LKA	**Sri Lanka**
22	PAK	**Pakistan**
23	XSA	**Rest of South Asia** *Afghanistan* (afg), *Bhutan* (btn), *Maldives* (mdv), *Nepal* (npl)
24	CAN	**Canada**
25	USA	**United States**
26	MEX	**Mexico**
27	XNA	**Rest of North America** *Bermuda* (bmu), *Greenland* (grl), *Saint Pierre & Miquelon* (spm)
28	ARG	**Argentina**
29	BOL	**Bolivia**
30	BRA	**Brazil**
31	CHL	**Chile**
32	COL	**Colombia**
33	ECU	**Ecuador**
34	PRY	**Paraguay**
35	PER	**Peru**
36	URY	**Uruguay**
37	VEN	**Venezuela, Republica Bolivariana de**
38	XSM	**Rest of South America** *Falkland Islands* (flk), *French Guiana* (guf), *Guyana* (guy), *Suriname* (sur)
39	CRI	**Costa Rica**
40	GTM	**Guatemala**
41	NIC	**Nicaragua**
42	PAN	**Panama**
43	XCA	**Rest of Central America** *Belize* (blz), *El Salvador* (slv), *Honduras* (hnd)
44	XCB	**Caribbean** *Anguilla* (aia), *Antigua & Barbuda* (atg), *Aruba* (abw), *Bahamas* (bhs), *Barbados* (brb), *Cayman Islands* (cym), *Cuba* (cub), *Dominica* (dma), *Dominican Republic* (dom), *Grenada* (grd), *Guadeloupe* (glp), *Haiti* (hti), *Jamaica* (jam), *Martinique* (mtq),

		Montserrat (msr), *Netherlands Antilles* (ant), *Puerto Rico* (pri), *Saint. Kitts & Nevis* (kna), *Saint Lucia* (lca), *Saint. Vincent and the Grenadines* (vct), *Trinidad & Tobago* (tto), *Turks and Caicos Islands* (tca), *British Virgin Islands*(vgb), *United States Virgin Islands* (vir)
45	AUT	**Austria**
46	BEL	**Belgium**
47	BGR	**Bulgaria**
48	CYP	**Cyprus**
49	CZE	**Czech Republic**
50	DNK	**Denmark**
51	EST	**Estonia**
52	FIN	**Finland**
53	FRA	**France**
54	DEU	**Germany**
55	GRC	**Greece**
56	HUN	**Hungary**
57	IRL	**Ireland**
58	ITA	**Italy**
59	LVA	**Latvia**
60	LTU	**Lithuania**
61	LUX	**Luxembourg**
62	MLT	**Malta**
63	NLD	**Netherlands**
64	POL	**Poland**
65	PRT	**Portugal**
66	ROU	**Romania**
67	SVK	**Slovakia**
68	SVN	**Slovenia**
69	ESP	**Spain**
70	SWE	**Sweden**
71	GBR	**United Kingdom**
72	NOR	**Norway**
73	CHE	**Switzerland**
74	XEF	**Rest of European Free Trade Area (EFTA)** *Iceland* (isl), *Liechtenstein* (lei)
75	ALB	**Albania**
76	BLR	**Belarus**
77	HRV	**Croatia**
78	RUS	**Russian Federation**
79	UKR	**Ukraine**
80	XEE	**Rest of Eastern Europe** *Moldova* (mda)
81	XER	**Rest of Europe** *Andorra* (and), , *Bosnia and Herzegovina* (bih), *Faroe Islands* (fro), *Gibraltar* (gib), *Macedonia* (mkd, *former Yugoslav Republic of*), *Monaco* (mco), *San Marino* (smr), *Serbia and Montenegro* (scg)
82	KAZ	**Kazakhstan**
83	KGZ	**Kyrgyz Republic**
84	XSU	**Rest of Former Soviet Union** *Tajikistan* (tjk), *Turkmenistan* (tkm), *Uzbekistan* (uzb)
85	ARM	**Armenia**
86	AZE	**Azerbaijan**
87	GEO	**Georgia**
88	IRN	**Iran**
89	TUR	**Turkey**
90	XWS	**Rest of Western Asia** *Bahrain* (bhr), *Iraq* (irq), *Israel* (isr), *Jordan* (jor), *Kuwait* (kwt), *Lebanon* (lbn), *West Bank and Gaza* (pse), *Oman* (omn), *Qatar* (qat), *Saudi Arabia* (sau), *Syrian Arab Republic* (syr), *United Arab Emirates* (are), *Republic of Yemen* (yem)
91	EGY	**Egypt**
92	MAR	**Morocco**
93	TUN	**Tunisia**
94	XNF	**Rest of North Africa** *Algeria* (dza), *Libyan Arab Jamahiriya* (lby)
95	NGA	**Nigeria**
96	SEN	**Senegal**

97	XWF	**Rest of Western Africa** *Benin* (ben), *Burkina Faso* (bfa), *Cape Verde* (cpv), *Côte d'Ivoire* (civ), *Gambia, The* (gmb), *Ghana* (gha), *Guinea* (gin), *Guinea-Bissau* (gnb), *Liberia* (lbr), *Mali* (mli), *Mauritania* (mrt), *Niger* (ner), *Saint Helena* (shn), *Sierra Leone* (sle), *Togo* (tgo)
98	XCF	**Central Africa** *Cameroon* (cmr), *Central African Republic* (caf), *Chad* (tcd), *Congo* (cog), *Equatorial Guinea* (gnq), *Gabon* (gab), *Sao Tome & Principe* (stp)
99	XAC	**South-Central Africa** *Angola* (ago), *Democratic Republic of the Congo* (cod, *formerly Zaïre*)
100	ETH	**Ethiopia**
101	MDG	**Madagascar**
102	MWI	**Malawi**
103	MUS	**Mauritius**
104	MOZ	**Mozambique**
105	TZA	**Tanzania**
106	UGA	**Uganda**
107	ZMB	**Zambia**
108	ZWE	**Zimbabwe**
109	XEC	**Rest of Eastern Africa** *Burundi* (bdi), *Comoros* (com), *Djibouti* (dji), *Eritrea* (eri), *Kenya* (ken), *Mayotte* (myt), *Réunion (reu)*, *Rwanda* (rwa), *Seychelles Islands* (syc), *Somalia* (som), *Sudan* (sdn)
110	BWA	**Botswana**
111	ZAF	**South Africa**
112	XSS	**Rest of South African Customs Union** *Lesotho (lso), Namibia (nam),Swaziland (swz)*

Table A9-2: Sectoral Concordance

1	PDR	**Paddy rice**
2	WHT	**Wheat**
3	GRO	**Cereal grains, n.e.s.**
4	V_F	**Vegetables and fruits**
5	OSD	**Oil seeds**
6	C_B	**Sugar cane and sugar beet**
7	PFB	**Plant-based fibers**
8	OCR	**Crops, n.e.s.**
9	CTL	**Bovine cattle, sheep and goats, horses**
10	OAP	**Animal products n.e.s.**
11	RMK	**Raw milk**
12	WOL	**Wool, silk-worm cocoons**
13	FRS	**Forestry**
14	FSH	**Fishing**
15	COA	**Coal**
16	OIL	**Oil**
17	GAS	**Gas**
18	OMN	**Minerals n.e.s.**
19	CMT	**Bovine cattle, sheep and goat, horse meat products**
20	OMT	**Meat products n.e.s.**
21	VOL	**Vegetable oils and fats**
22	MIL	**Dairy products**
23	PCR	**Processed rice**
24	SGR	**Sugar**
25	OFD	**Food products n.e.s.**
26	B_T	**Beverages and tobacco products**
27	TEX	**Textiles**
28	WAP	**Wearing apparel**
29	LEA	**Leather products**
30	LUM	**Wood products**
31	PPP	**Paper products, publishing**
32	P_C	**Petroleum, coal products**
33	CRP	**Chemical, rubber, plastic products**
34	NMM	**Mineral products n.e.s.**
35	I_S	**Ferrous metals**
36	NFM	**Metals n.e.s.**
37	FMP	**Metal products**
38	MVH	**Motor vehicles and parts**
39	OTN	**Transport equipment n.e.s.**
40	ELE	**Electronic equipment**
41	OME	**Machinery and equipment n.e.s.**
42	OMF	**Manufactures n.e.s.**
43	ELY	**Electricity**
44	GDT	**Gas manufacture, distribution**
45	WTR	**Water**
46	CNS	**Construction**
47	TRD	**Trade**
48	OTP	**Transport n.e.s.**
49	WTP	**Sea transport**
50	ATP	**Air transport**
51	CMN	**Communication**
52	OFI	**Financial services n.e.s.**
53	ISR	**Insurance**
54	OBS	**Business services n.e.s.**
55	ROS	**Recreation and other services**
56	OSG	**Public administration and defense, education, health services**
57	DWE	**Dwellings**

Figures

Figure 1: Production structure nesting

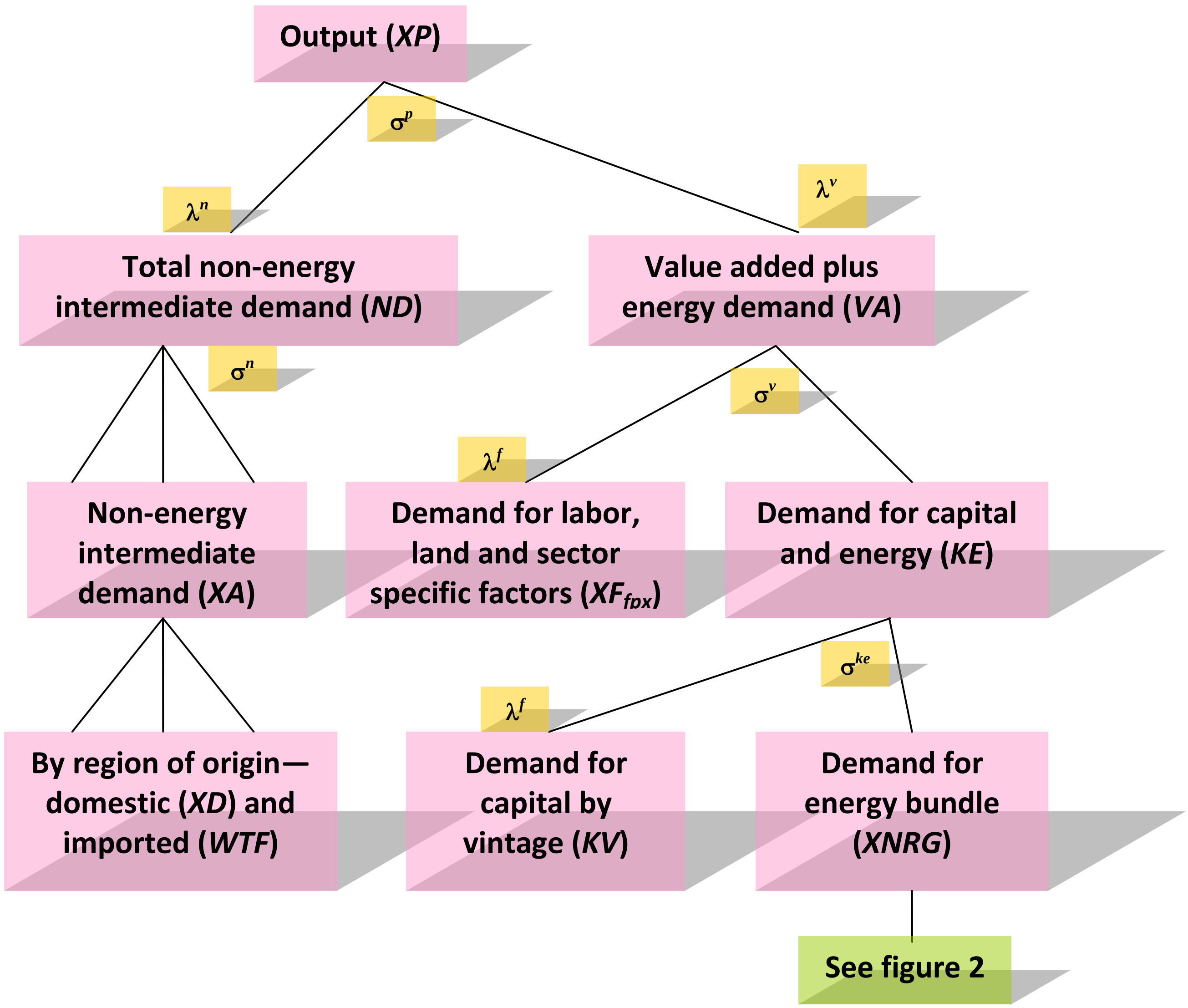

Figure 2: Energy nesting

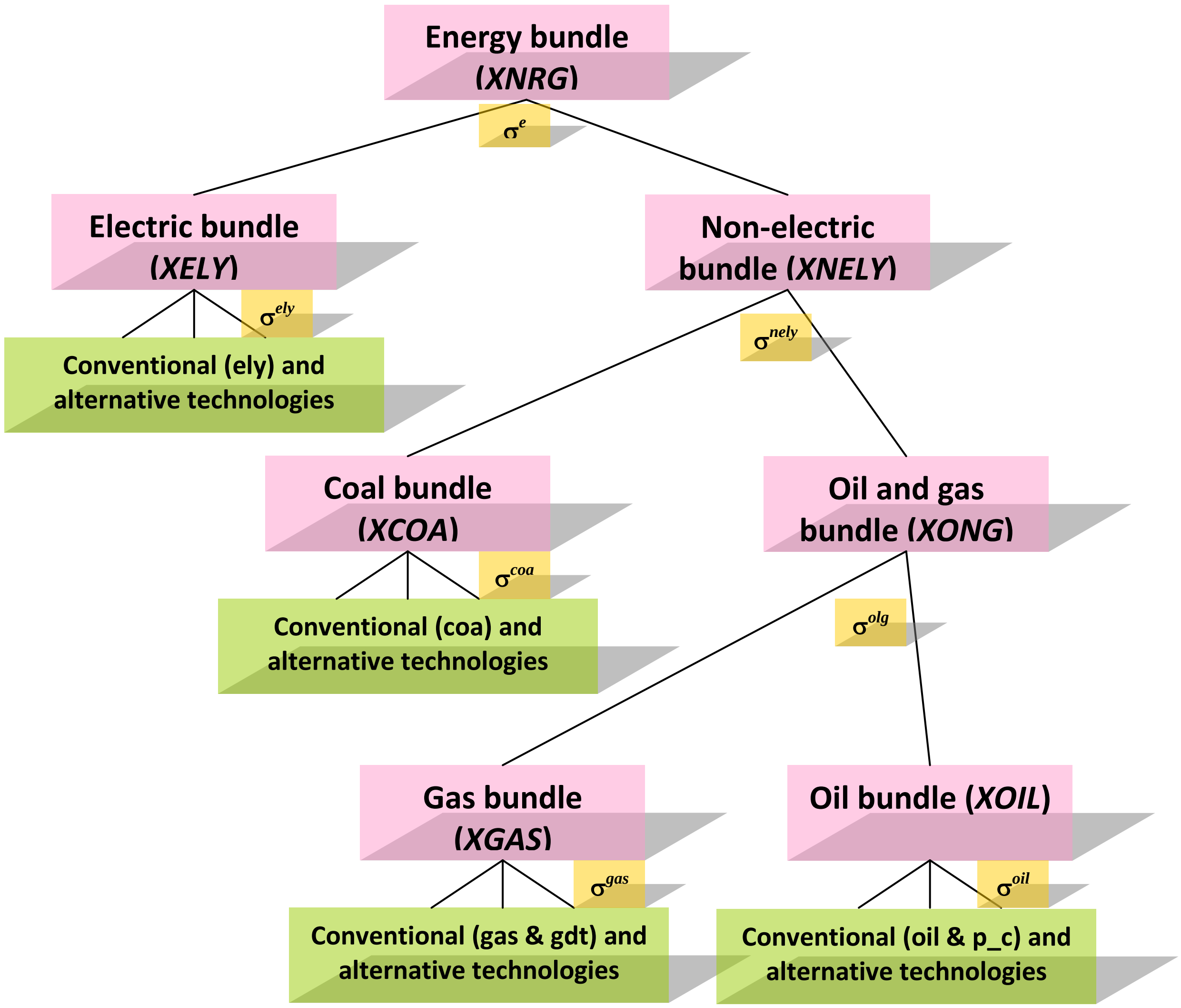

Figure 3: Output, supply and trade

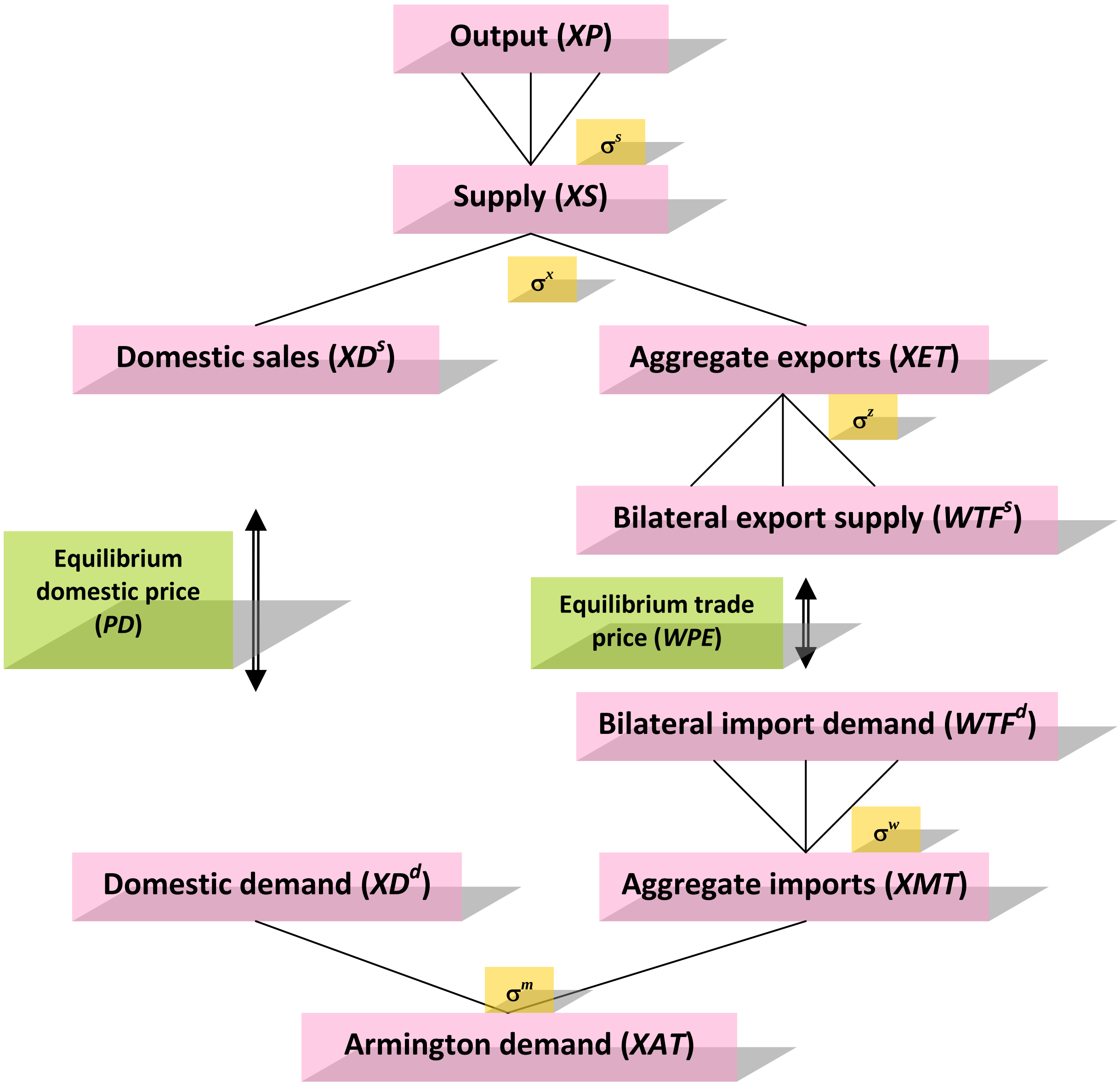

Figure 4: Domestic demand nesting

The allocation of national income across expenditure categories is determined by the closure rules. By default all net factor income accrues to households. Households allocate their disposable income between savings and expenditures on goods and services. The savings function depends on a number of factors including demographic variables. Public expenditures are fixed as a share of nominal GDP. Investment expenditures are constrained by available savings—household, public (normally set at zero) and foreign. The default household demand specification is the CDE, but the model includes the ELES and AIDADS as well.